Business Cycle Facts: A Short Note

Macroeconomics: Economic Cycles, Frictions and Policy

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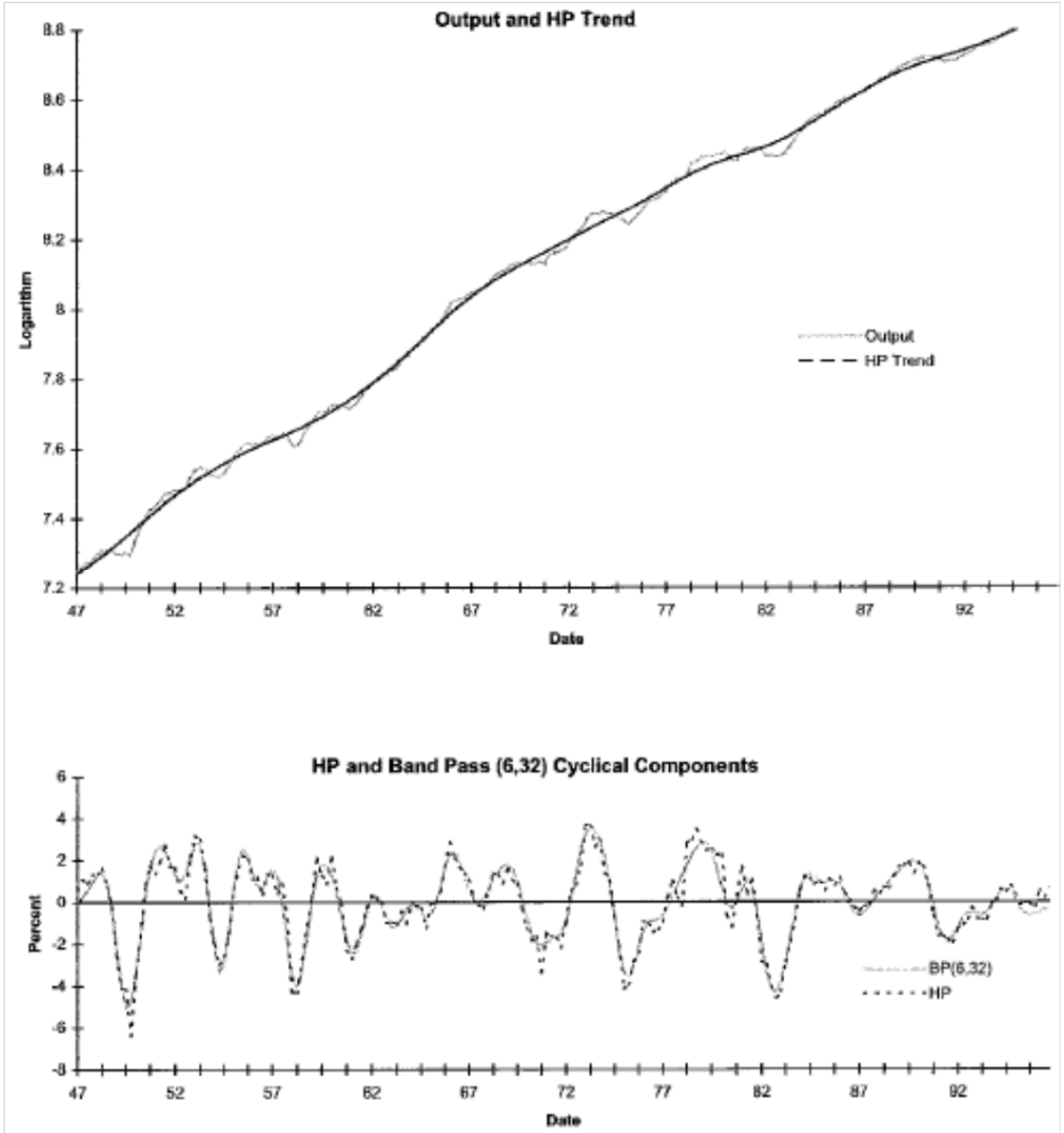
Outline

1. Economic trends and cycles

2. Business cycle facts: volatility, comovement and persistence

Economic trends and cycles



How do we separate the cycle from the trend?

- Hodrick & Prescott (1980) detrended their variables using a procedure now widely known as the HP filter
- Cyclical output y_t^c is defined as current output y_t less a measure of trend output y_t^g,

$$y_t^c = y_t - y_t^g. \tag{1}$$

  where y_t^g is a weighted average of past, current and future observations
- More formally, trend output is found by solving the following program:

$$\min_{\{y_t^g\}_{t=0}^T} \sum_{t=0}^{T} [y_t - y_t^g]^2, \tag{2}$$

s.t.:

$$\sum_{t=2}^{T-1} [(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2 \leq \lambda. \tag{3}$$

- The Lagrangian for this problem is thus:

$$\mathcal{L} = \sum_{t=0}^{T} [y_t - y_t^g]^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2 \tag{4}$$

- λ penalizes for the "roughness" of the series: $\lambda = 0 \Rightarrow y_t^g = y_t$ and $\lambda \to \infty \Rightarrow$ linear trend

How do we separate the cycle from the trend?

$$\mathcal{L} = \sum_{t=0}^{T} [y_t - y_t^g]^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2$$

- The FOC for the problem for y_t^g are:

$$[y_t^g]: \quad y_t - y_t^g = \lambda[y_{t+2}^g - 4y_{t+1}^g + 6y_t^g - 4y_{t-1}^g + y_{t-2}^g], \quad t = 3, \ldots, T-2$$

- Notice that the FOCs will be slightly different for $t = 1, 2, T-1, T$. you should derive these at home
- Doing so, and letting $\mathbf{y^g} \equiv [y_1^g, \ldots, y_T^g]'$ be a $T \times 1$ vector, and $\mathbf{y} - \mathbf{y^g} \equiv [y_1 - y_1^g, \ldots, y_T - y_T^g]'$, we can write the T FOCs compactly as:

$$\mathbf{y} - \mathbf{y^g} = \lambda \mathbf{F} \mathbf{y^g} \tag{5}$$

  where $\mathbf{F}$ is a $T \times T$ matrix with special properties

- Which means:

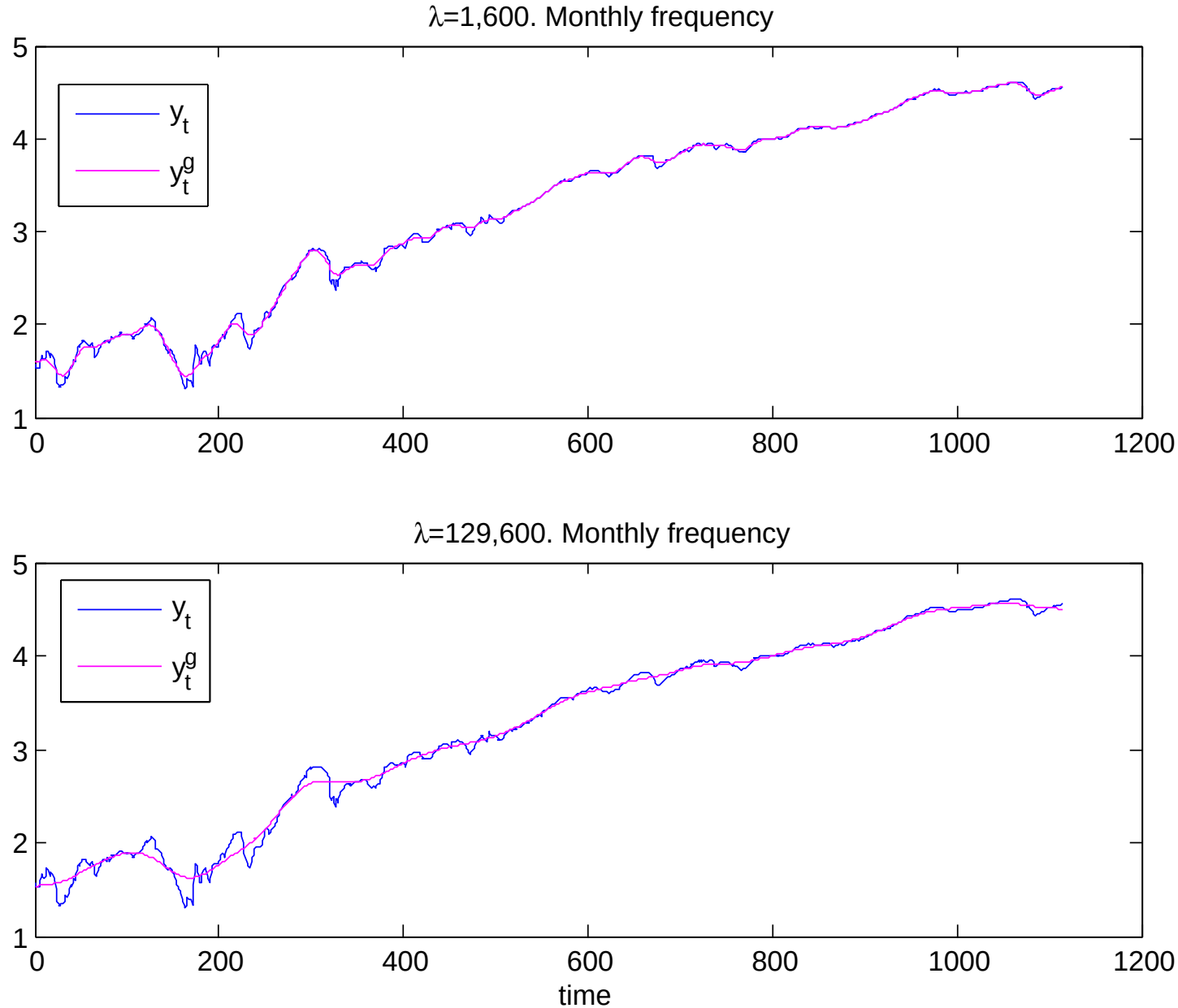
$$\mathbf{y^g} = (I_T + \lambda \mathbf{F})^{-1} \mathbf{y} \tag{6}$$

- and $\mathbf{y^c} = \mathbf{y} - \mathbf{y^g}$

This is how $\mathbf{F}$ looks like

$$\mathbf{F} = \begin{pmatrix} 1 & -2 & 1 & 0 & \cdots & & & & & 0 \\ -2 & 5 & -4 & 1 & 0 & \cdots & & & & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & \cdots & & & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & \cdots & & 0 \\ \vdots & & & & & & & & & \vdots \\ 0 & \cdots & & & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & \cdots & & & & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & \cdots & & & & & 0 & -2 & 5 & -4 & 1 \\ 0 & \cdots & & & & & & 0 & 1 & -2 & 1 \end{pmatrix}_{T \times T} \tag{7}$$

HP-filter generated trends for different λs



How do we separate the cycle from the trend?

- The HP filter is designed to eliminate stochastic components with periodicities greater than 32 quarters.
- The value of λ depends on the periodicity of your data. Common values used are:

$$\lambda = \begin{cases} 129600 & \text{for monthly data} \\ 1600 & \text{for quarterly data} \\ 6 & \text{for yearly data} \end{cases}$$

- For information look at the paper by Ravn & Uhlig (2002) REStat
- There are other filters available, see for instance the Band-pass filter developed by Baxter & King (1995) REStat. The main objective is the same as what you achieve with the HP filter

Business cycle facts

Table 1
Business cycle statistics for the US Economy

	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	−0.35
A	0.98	0.54	0.74	0.78

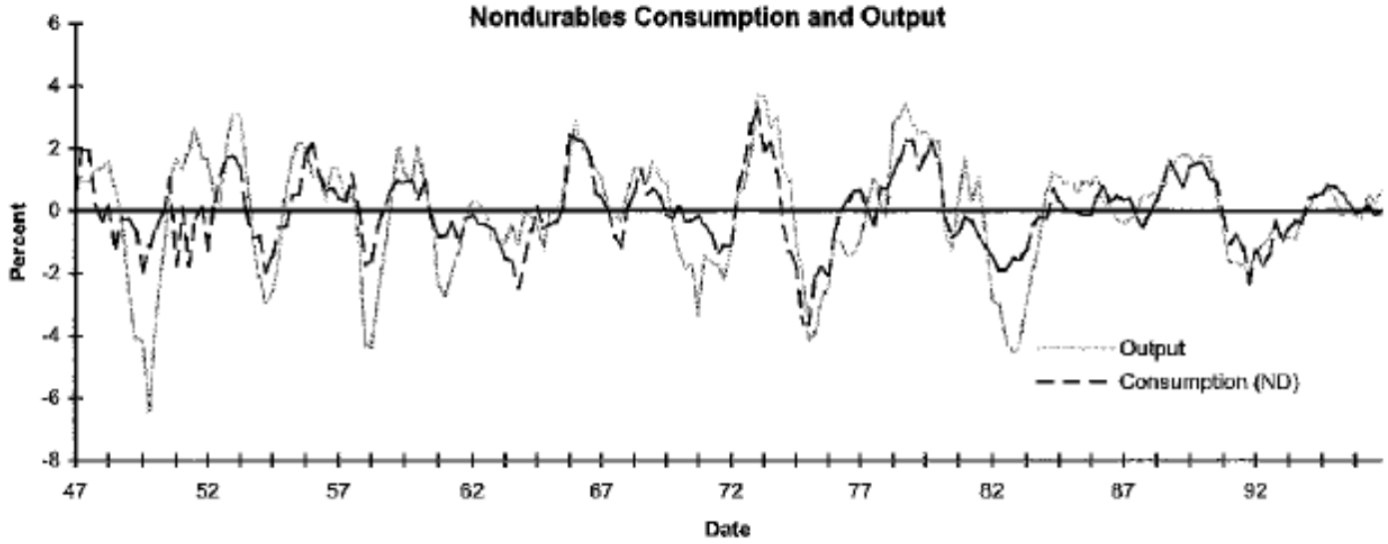
[a] All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. Data sources are described in Stock and Watson (1999), who created the real rate using VAR inflation expectations. Our notation in this table corresponds to that in the text, so that Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity.

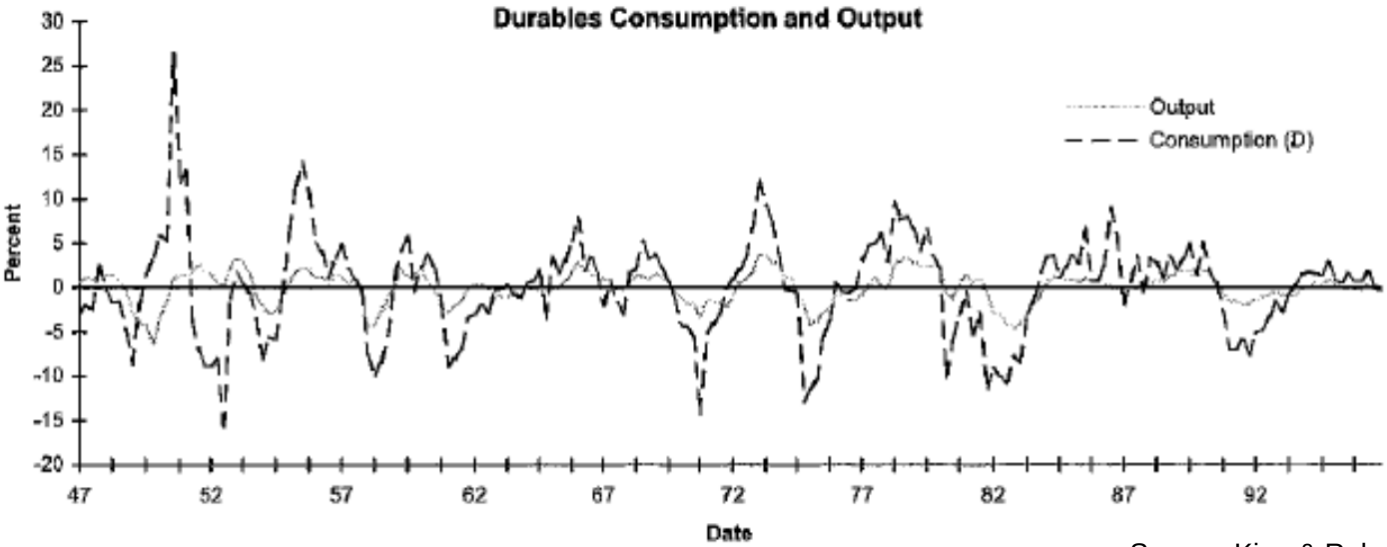
Source: King & Rebelo (1999)

Business cycle facts: volatility

1. $\sigma(C_{nd}) < \sigma(Y)$
2. $\sigma(C_d) > \sigma(Y)$
3. $\sigma(I) = 3\sigma(Y)$
4. $\sigma(G) < \sigma(Y)$
5. $\sigma(\text{hours}) \approx \sigma(Y)$
6. $\sigma(K) < \sigma(Y)$, but $\sigma(K_{ut}) > \sigma(Y)$
7. $\sigma(Y/N) < \sigma(Y)$
8. $\sigma(W/P) < \sigma(Y)$
9. $\sigma(N) \approx \sigma(Y)$ and $\sigma(\text{hours/worker}) < \sigma(Y) \Rightarrow$ most of the cyclical variation in total hours worked stems from changes in employment

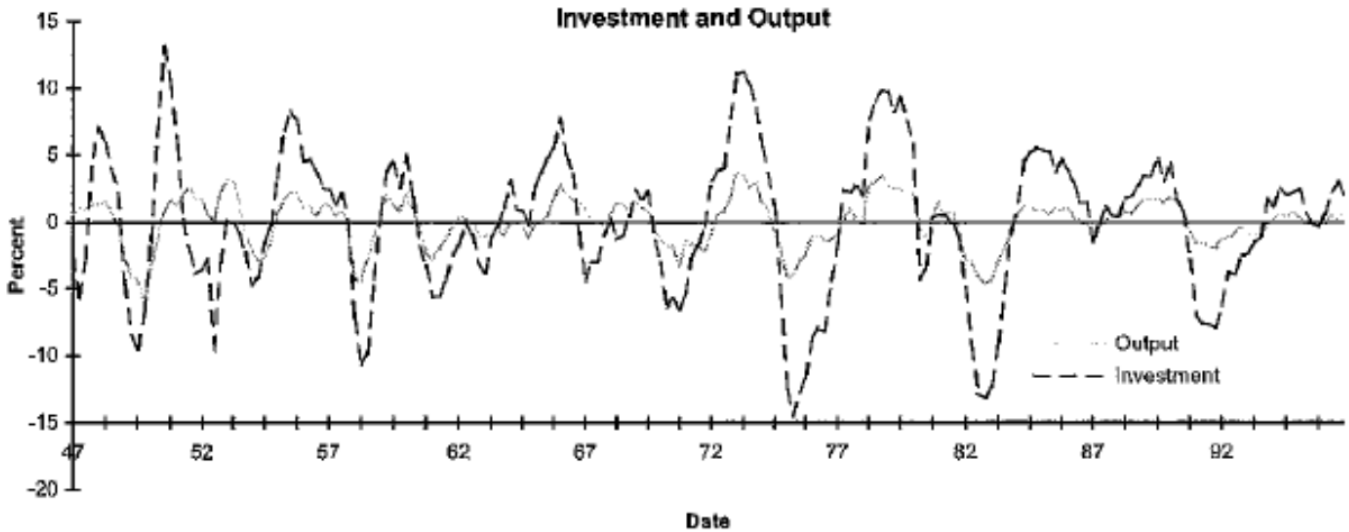
Business cycle facts

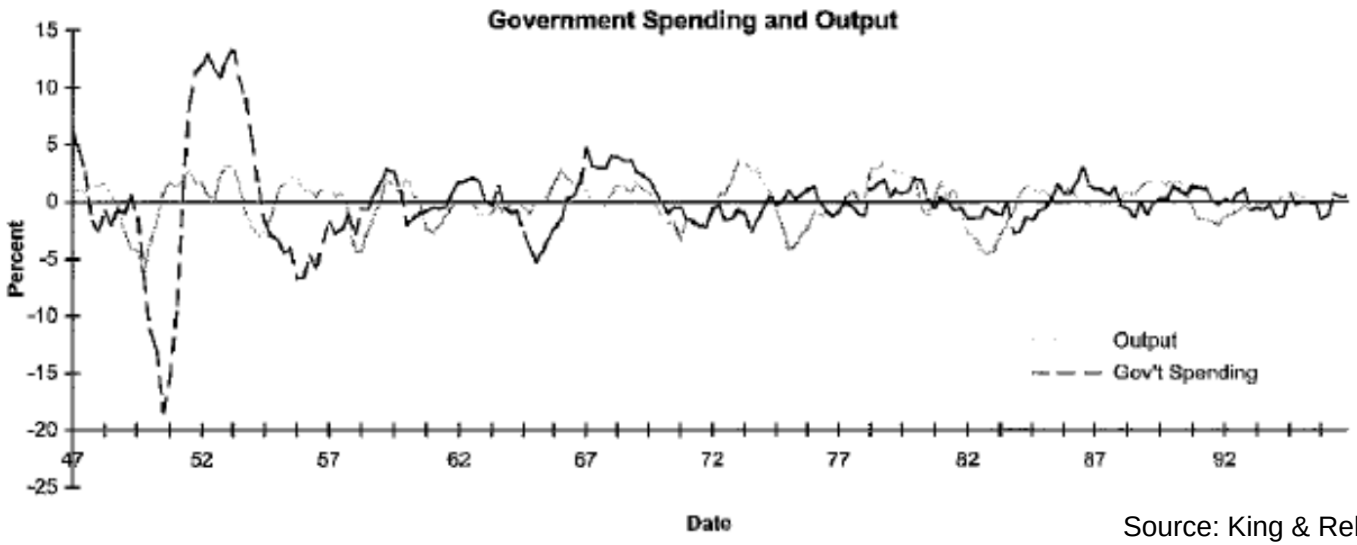




Source: King & Rebelo (1999)

Business cycle facts





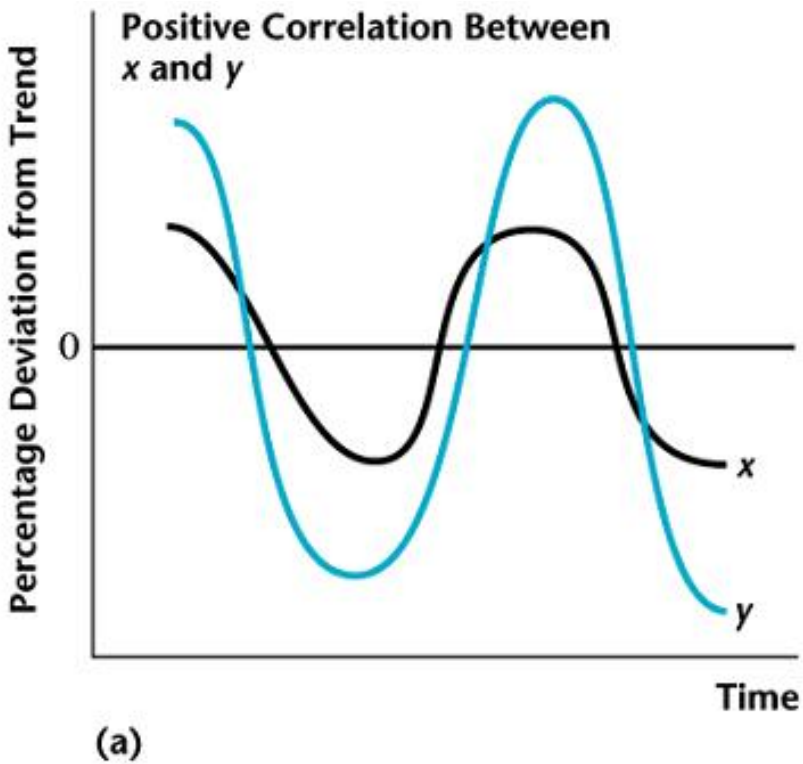
Source: King & Rebelo (1999)

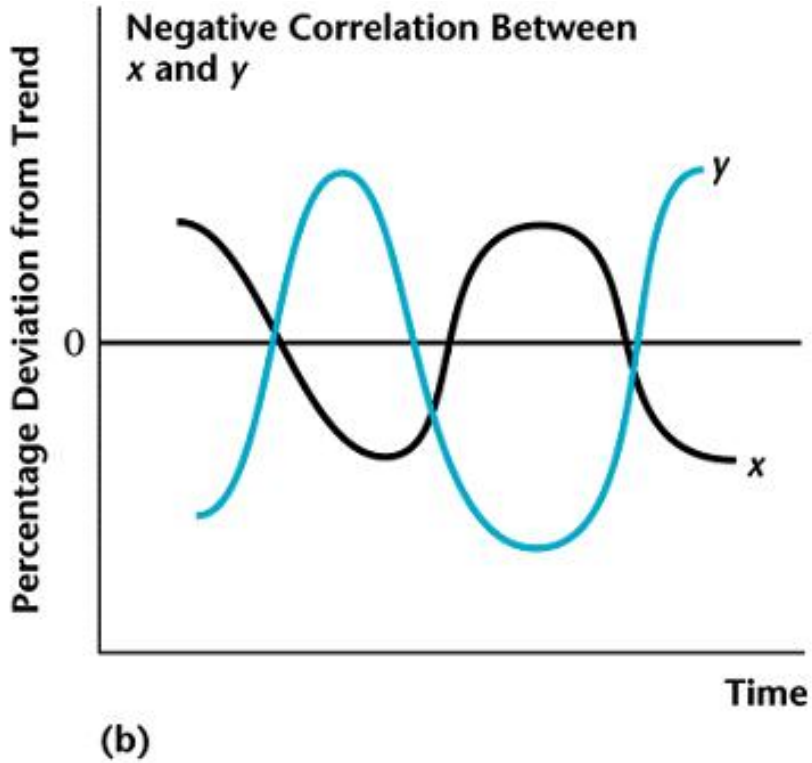
Business cycle facts: co-movement and persistence

- Most macro variables are pro-cyclical: they have a strong positive contemporaneous correlation with output
- The obvious exception is unemployment, which is counter-cyclical
- Three series are essentially acyclical: wages, government expenditure and capital
- All macroeconomic aggregates display substantial persistence; the first-order serial correlation for most detrended quarterly variables is on the order of 0.9

Co-movement

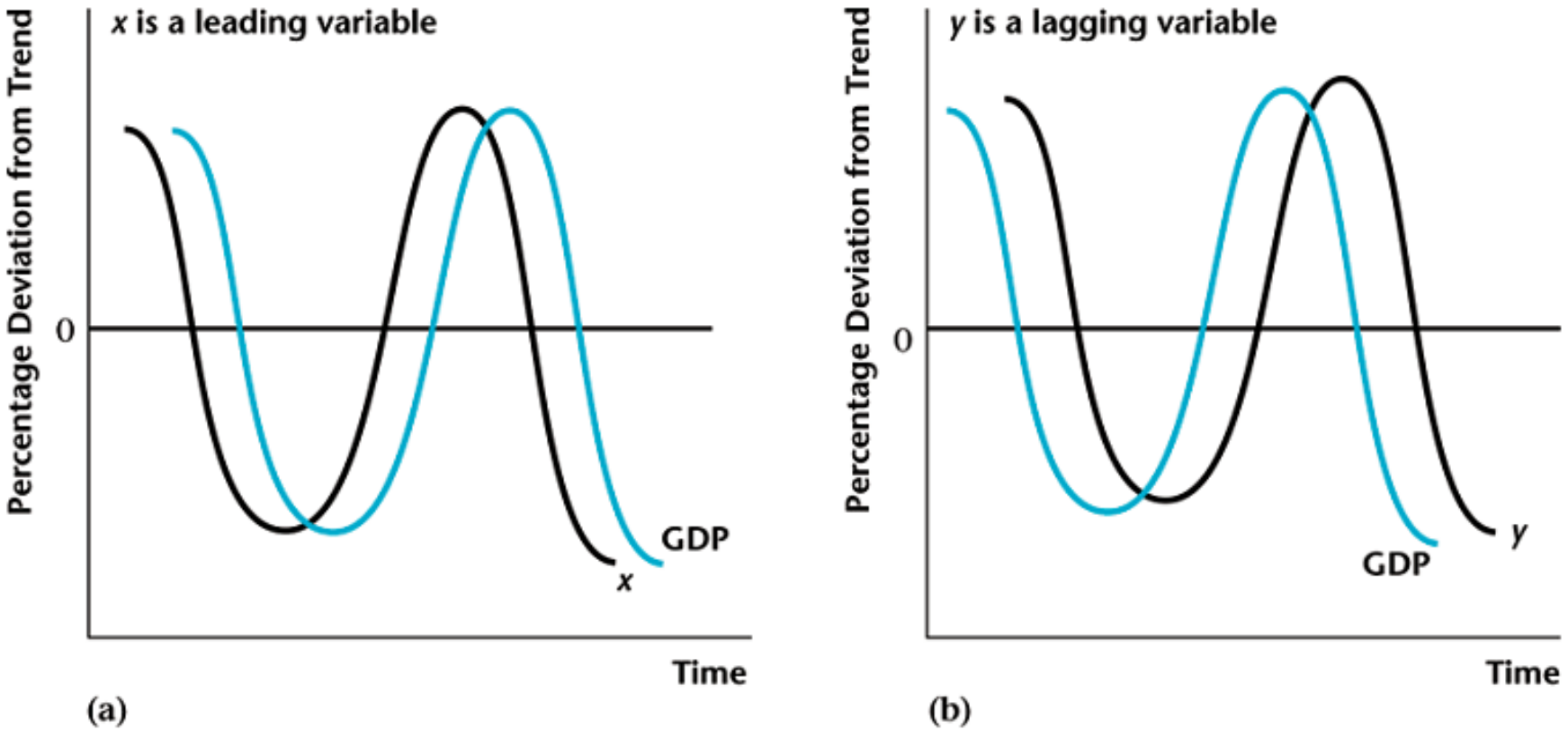
- Panel A: non-durable consumption
- Panel B: unemployment





Leading/Lagging variables

- Panel A: residential investment; inventories
- Panel B: inflation; nominal interest rates



References

- King, R. and S. Rebelo. 1999. "Resuscitating Real Business Cycle Models", in John Taylor and Michael Woodford, eds., *Handbook of Macroeconomics* vol. 1A. Amsterdam: Elsevier. Chapter 14.
- Stock, J. H. and M. W. Watson "Business Cycle Fluctuations in US Macroeconomic Time Series" in John Taylor and Michael Woodford, eds., *Handbook of Macroeconomics* vol. 1A. Amsterdam: Elsevier. Chapter 1.