Economic Growth

Macroeconomics: Economic Cycles, Frictions and Policy

Alejandro Riaño

University of Nottingham, GEP, CFCM, and CESifo

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Outline

The Solow growth model

- Infinite horizon, continuous time
- Single, homogeneous consumption good
- Aggregate production function:

$$Y = F(K, L) = K^{\alpha}(AL)^{1-\alpha}, \quad \alpha \in (0, 1) \tag{1}$$

  where AL is effective units of labor
- Output-per-worker:

$$y \equiv \frac{Y}{L} = A^{1-\alpha}\left(\frac{K}{L}\right)^{\alpha} = A^{1-\alpha}k^{\alpha} \tag{2}$$

  lower-case variables (output, capital, etc.) are in per worker terms
- To understand the evolution of output per worker we need to know what happens to capital per worker
- Population (=labor force) grows exogenously at a constant rate n:

$$L(t) = L(0)e^{nt} \Leftrightarrow \frac{\dot{L}}{L} = n \tag{3}$$

- For any variable x, let $\dot{x} \equiv \frac{dx}{dt}$

Capital accumulation

- Physical capital grows through investment; every period, a constant share $\delta \in (0,1)$ of the installed capital depreciates
- Consumers are myopic: they save a constant fraction $s \in (0,1)$ of their income no matter what $\rightarrow$ aggregate saving $S = sY$
- No government and closed economy. Therefore:

$$GDP = Y = C + I + \underbrace{G}_{=0} + \underbrace{(X - M)}_{=0}$$

$$Y - C = I \Rightarrow S = I \tag{4}$$

- The law-of-motion for the capital stock is then:

$$\dot{K} = sY - \delta K \tag{5}$$

- Assume for the time being that A is constant over time
- Divide both sides of (5) by L:

$$\frac{\dot{K}}{L} = sA^{1-\alpha}k^{\alpha} - \delta k \tag{6}$$

Capital accumulation

- Note that:

$$\dot{k} \equiv \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{K}{L}\right) = \frac{\dot{K}L - \dot{L}K}{L^2} = \frac{\dot{K}}{L} - \frac{K}{L}\frac{\dot{L}}{L}$$

$$\dot{k} = \frac{\dot{K}}{L} - kn \tag{7}$$

- Substitute (7) into (6) and divide by k:

$$\boxed{\frac{\dot{k}}{k} \equiv g_k = sA^{1-\alpha}k^{\alpha-1} - (n+\delta)} \tag{8}$$

where g_k is the rate of growth of capital per worker

Balanced growth path

- A balanced-growth path (BGP) as a situation in which all endogenous variables in the model are growing at constant rates (these do not need to be all the same)
- In the BGP $\frac{\mathrm{d}}{\mathrm{d}t}(\dot{k}/k) = 0$
- For this to be the case, we need that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[sA^{1-\alpha}k^{\alpha-1}\right] = 0 \tag{9}$$

  in (8)
- Taking logs and differentiating w.r.t. time reveals that:

$$\dot{k}/k = \dot{A}/A = 0 \tag{10}$$

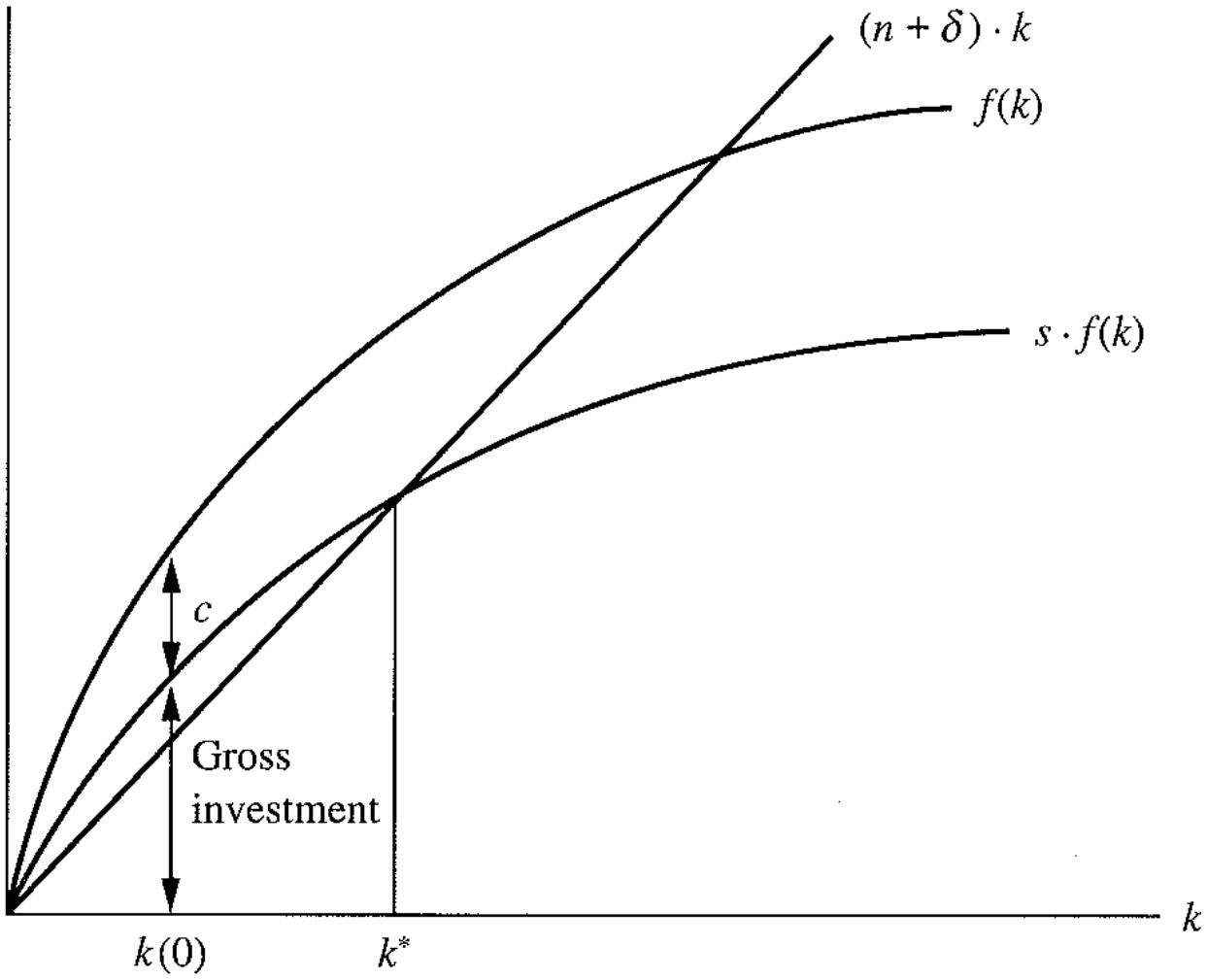
- Without technological progress, the BGP is characterized by a steady state in which capital per worker is constant at a level k^*
- We can solve for k^* by setting $\dot{k}/k = 0$ in (8):

$$k^* = A\left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}} \tag{11}$$

- GDP per capita is also constant in BGP:

$$y^* = A^{1-\alpha}\left(k^*\right)^{\alpha} = A\left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} \tag{12}$$

Basic Solow diagram



Transition dynamics

- We have established that capital per worker in the Solow model converges to a steady state
- What happens to k outside of the steady state? Go back to equation (8) and differentiate w.r.t. k:

$$\frac{\mathrm{d}g_k}{\mathrm{d}k} = -(1-\alpha)sA^{1-\alpha}k^{\alpha-2} < 0 \tag{13}$$

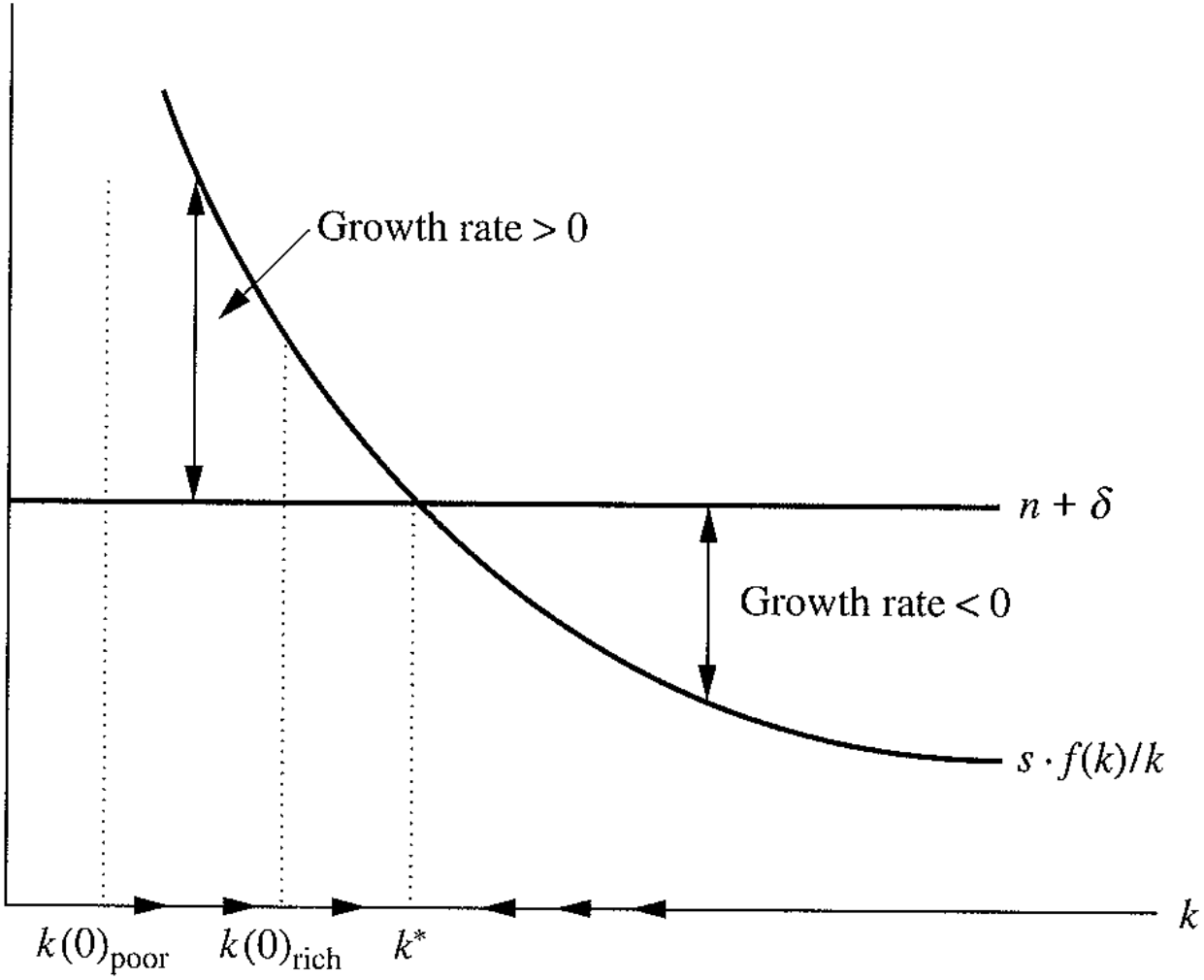
- The growth rate of capital per worker is decreasing in k. The growth rate of k and y is higher in poorer economies
- Also note that

$$\lim_{k\to 0} sA^{1-\alpha}k^{\alpha-1} \to \infty \tag{14}$$

$$\lim_{k\to \infty} sA^{1-\alpha}k^{\alpha-1} \to 0 \tag{15}$$

- This means that $sA^{1-\alpha}k^{\alpha-1}$ intersects the line $(n+\delta)$ once and only once in the positive quadrant $\to$ there is a unique and strictly positive level of steady state capital per worker
- No matter what the initial level of k is, the economy always converges to k^*

Transition dynamics



$$g_k = \begin{cases} > 0, & \text{if } \; k < k^* \\ < 0, & \text{if } \; k > k^*. \end{cases}$$

Implications

- Saving rates, population growth and the level of productivity affect the steady state level of capital per worker and output
- But not their growth rates (=0 in the long-run)!
- Capital accumulation alone cannot generate sustainable growth
- If $\dot{A}/A \equiv g_A > 0 \rightarrow g_k = g_y = g_A$ in the BGP
- But productivity growth occurs out of thin air!
- Since production has constant returns to scale

$$Y = \frac{\partial F}{\partial K} K + \frac{\partial F}{\partial L} L \qquad \text{(by Euler's theorem)}$$

$$Y = rK + wL \tag{16}$$

  GDP is all used to pay factors of production

- When $g_A > 0$, changes in s and n will have <u>transitory</u> effects on the growth rate of output along the transition path to a new steady state

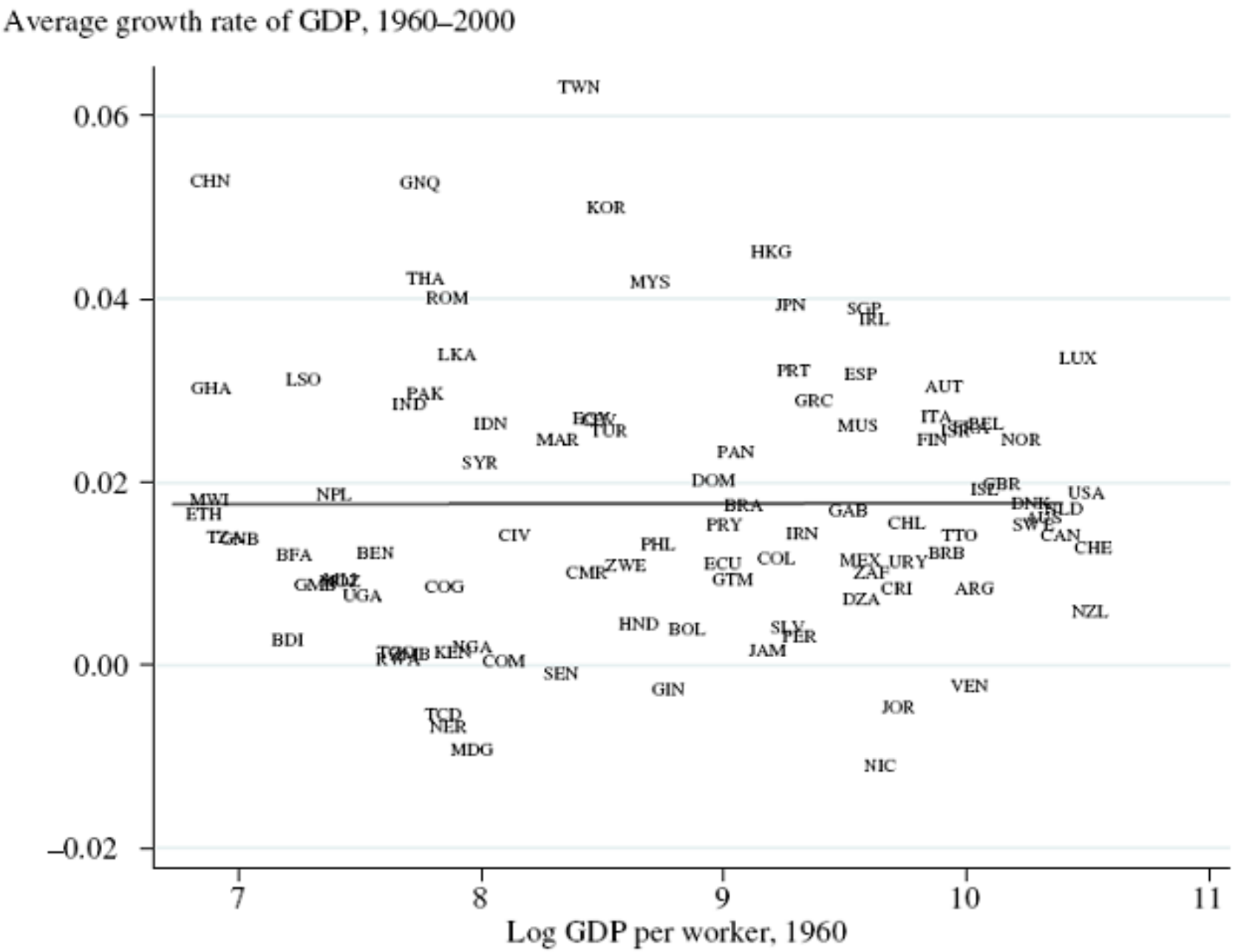
The Kaldor growth facts

- We want theoretical models to match reality as closely as possible
- In particular, researchers over the last 50 years have focused on the following set of empirical regularities (aka stylized facts), first documented by Nicholas Kaldor in 1963:
  1. Per capita output grows over time, and its growth rate does not tend to diminish ✓ when $g_A > 0$
  2. Physical capital per worker has also grown at a sustained rate ✓ when $g_A > 0$
  3. The real rate of return on capital has been stable ✓
  4. The ratio of capital to output has also been stable ✓
  5. Capital and labor have captured stable shares of national income ✓ with Cobb-Douglas technology
  6. The growth rate of output per worker differs substantially across countries — not in the simple model

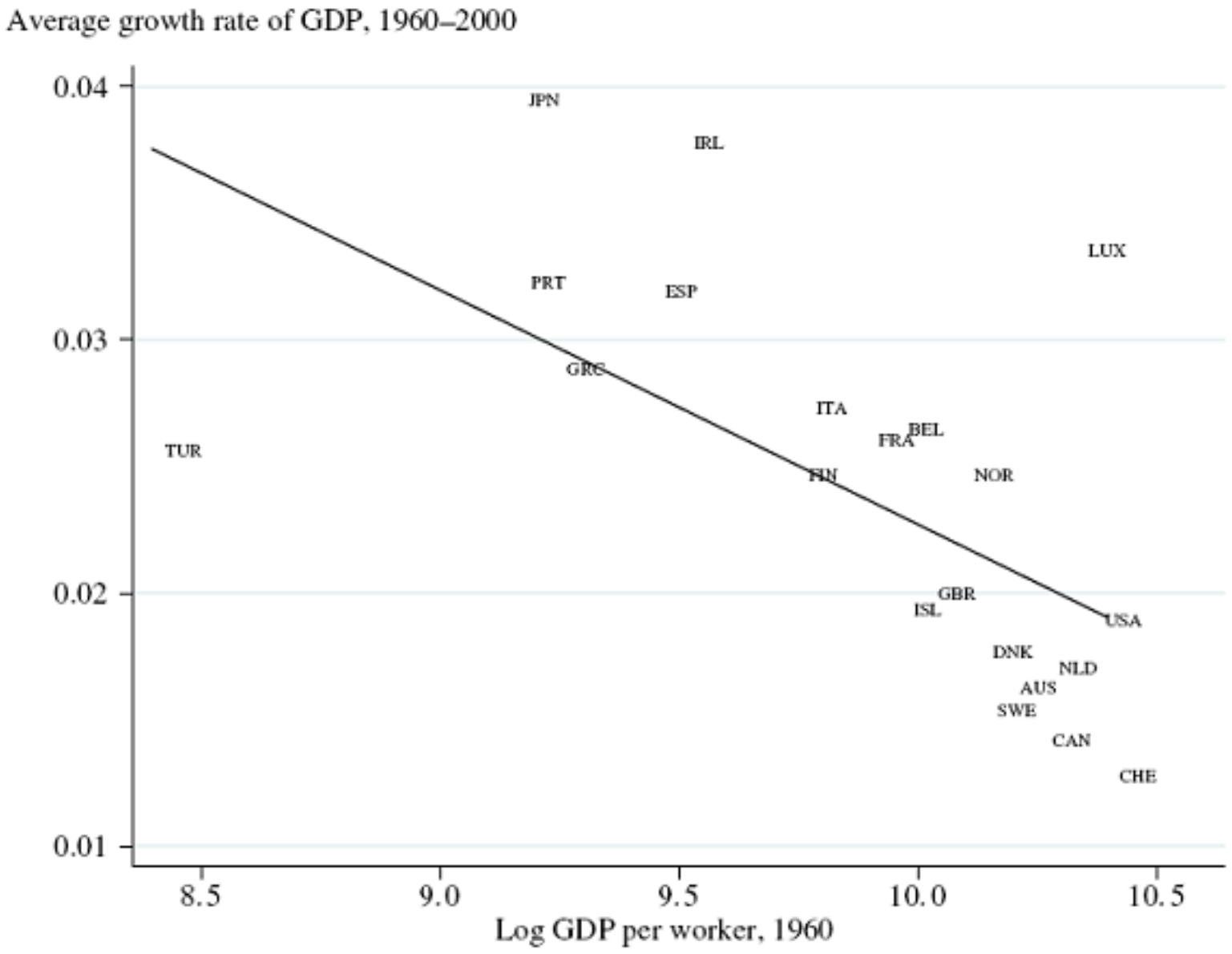
Absolute and conditional convergence

- If all countries in the world shared the same $(s, n, \delta) \rightarrow$ all countries will eventually reach the same level of capital per worker in steady state
- If you compare two countries with different levels of capital per worker, say $k^1 < k^2 \rightarrow g_k^1 > g_k^2$
- Poor countries would tend to grow faster than rich countries, without conditioning on any other characteristics of the economies. This hypothesis is called absolute convergence
- If we allow for heterogeneity in parameters across countries $\rightarrow$ different countries would reach different steady states. The testable hypotheses that follow from conditional convergence are that:
  - each economy converges to its own steady state
  - the speed of this convergence is inversely related to the distance from steady state

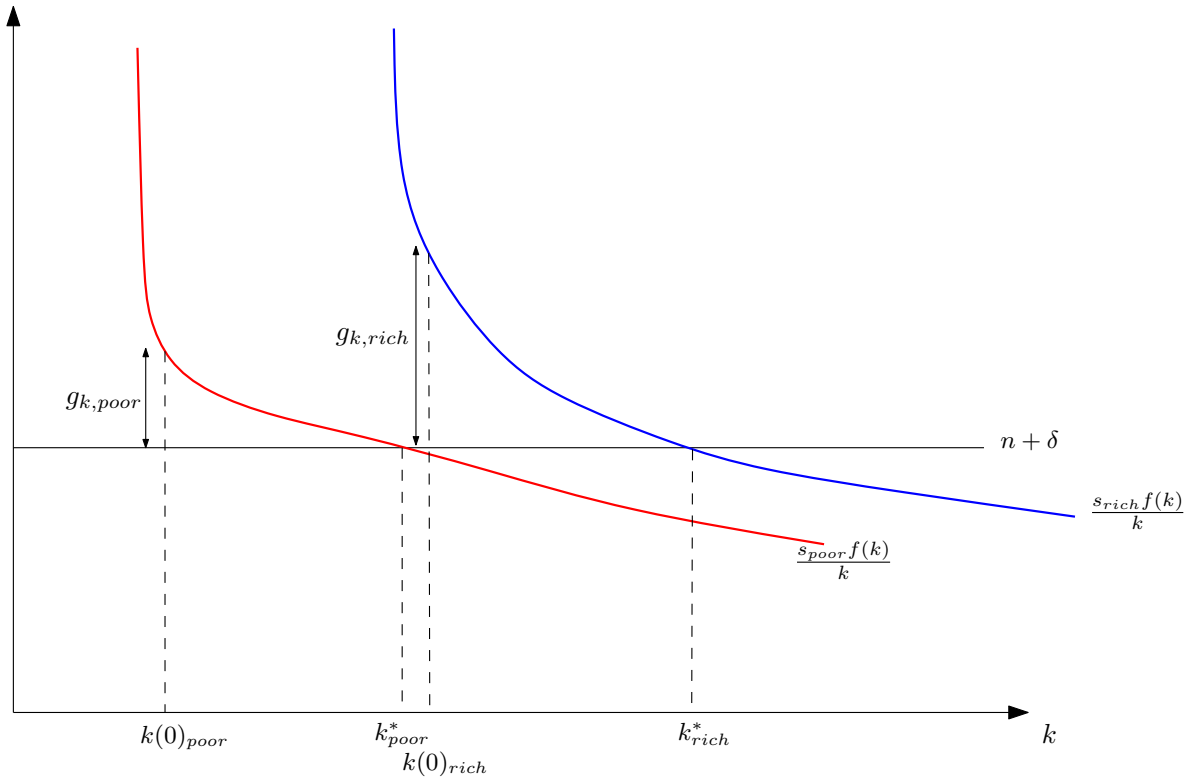
(Lack of) Unconditional convergence



Convergence within OECD countries

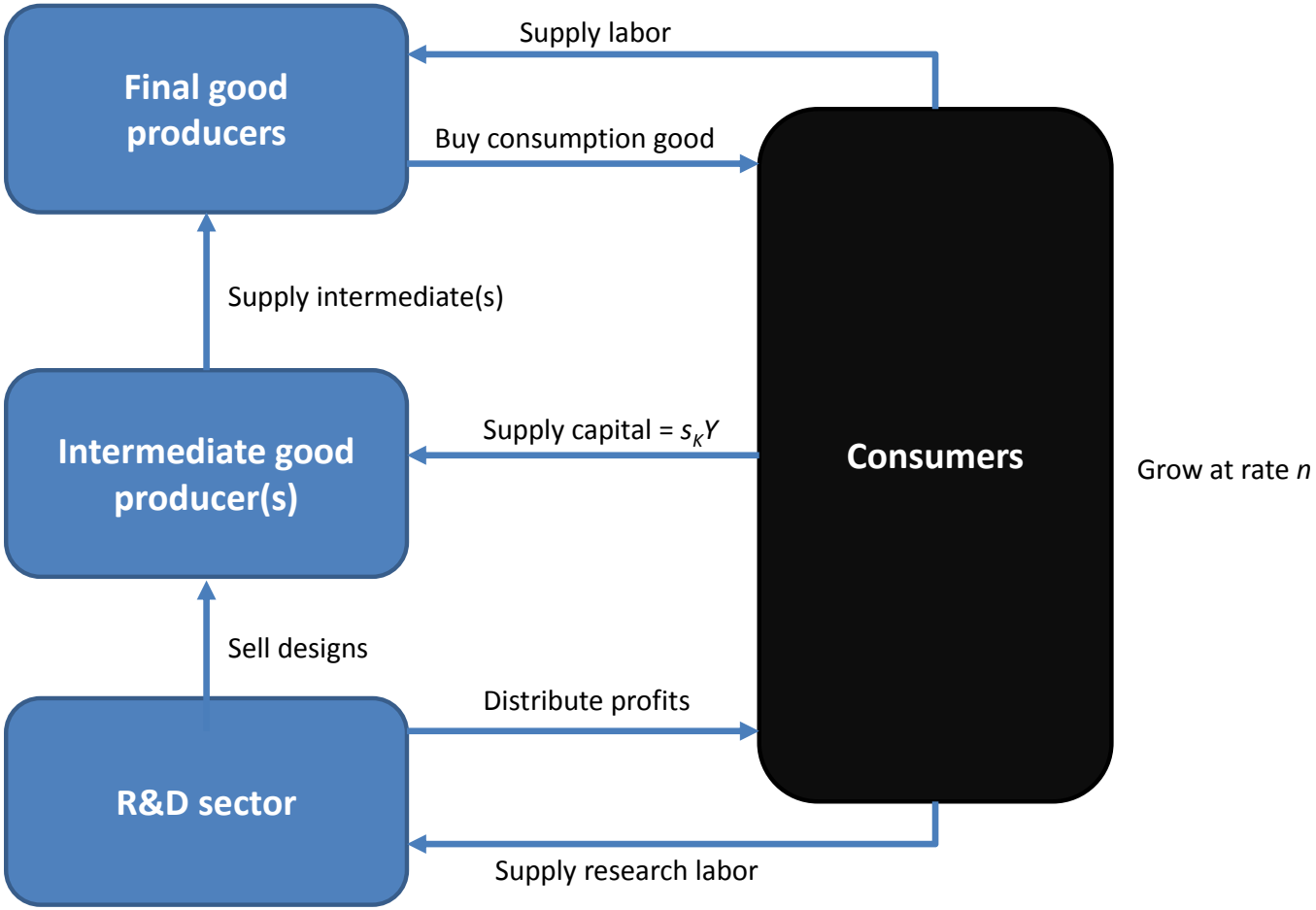


Illustrating conditional convergence



- Suppose that a rich and poor country are identical except on their initial level of capital $k(0)$ and their savings rate, s
- You can see that even though $k(0)_{\text{rich}} > k(0)_{\text{poor}}$, $g_{k,\text{rich}} > g_{k,\text{poor}} \rightarrow$ we would not observe unconditional convergence in the data
- However, if we were to control for the difference in savings rate, we should observe a negative correlation between g_k and $k(0)$

Romer's (1990) model of horizontal innovation



- Population grows at constant rate n
- Physical capital accumulates as usual: $\dot{K} = s_K Y - \delta K$. Capital is used in the production of intermediate goods only
- Labor can be used for production or research: $L_Y + L_A = L$

The model

- There is a large number of perfectly-competitive firms producing a final consumption good using the following production function:

$$Y = L_Y^{1-\alpha} \sum_{j=1}^{A} (X_j)^{\alpha}, \quad \alpha \in (0,1), \tag{17}$$

  where X_j denotes the quantity of input j and A denotes the total number of intermediates available in the economy at a given point in time

- Notice that the marginal product of intermediate j is independent of the quantity of intermediate $j' \rightarrow$ new products do not make old ones obsolete — horizontal innovation

- Assume that $X_j = X$, i.e. all firms use the same quantity of each intermediate good, then:

$$Y = L_Y^{1-\alpha} A X^{\alpha} = L_Y^{1-\alpha} (AX)^{\alpha} A^{1-\alpha}. \tag{18}$$

- The production function exhibits CRS in L_Y and AX, the total quantity of intermediates

- However, for given L_Y and AX, Y increases with A!

Final-good producers

- We normalize the price of Y to 1. Thus, the profit maximization problem of final-good producers is:

$$\max_{L_Y, X_j} \underbrace{L_Y^{1-\alpha} \sum_{j=1}^{A} (X_j)^{\alpha}}_{\text{revenue=price}\times\text{quantity}} - \underbrace{wL_Y - \sum_{j=1}^{A} P_j X_j}_{\text{costs}} \tag{19}$$

- The FOC of the problem are given by:

$$[L_Y]: \ (1-\alpha)\left(\frac{Y}{L_Y}\right) = w \tag{20}$$

$$[X_j]: \ \alpha L_Y^{1-\alpha} X_j^{\alpha-1} = P_j \tag{21}$$

- From (21) we have: $X_j = L_Y \left(\frac{\alpha}{P_j}\right)^{\frac{1}{1-\alpha}}$. This is the total demand for intermediate good j in the economy, X_j

The intermediate good sector

- 1 unit of X_j requires 1 unit of capital to be produced. There is a fixed cost associated with the purchase of the design for intermediate j from its inventor
- Profit-maximization for producer of intermediate j is:

$$\pi_j = \max_{X_j} \underbrace{\alpha L_y^{1-\alpha} X_j^{\alpha}}_{=p_j(X_j)X_j} - rX_j \tag{22}$$

- Taking the FOC: $p_j = p = \frac{r}{\alpha}$ and $\pi_j = \pi = \alpha(1-\alpha)\frac{Y}{A}$
- All capital in the economy is used to produce the A intermediate goods in the economy:

$$\sum_{j=1}^{A} X_j = K,$$

$$X = \frac{K}{A}$$

- Final output can be written as $Y = L_Y^{1-\alpha} A \left(\frac{K}{A}\right)^{\alpha} = K^{\alpha}(AL_Y)^{1-\alpha}$
- Aggregate output exhibits CRS on K and L_Y but increasing-returns to scale on K, L_Y and A altogether, which is what we had shown before

R&D sector

- Idea = design for a new intermediate good j
- Production function for ideas:

$$\dot{A} = \overline{\theta} L_A \tag{23}$$

$$\boxed{\dot{A} = [\theta L_A^{\lambda-1} A^{\phi}] L_A, \quad \lambda \in (0,1), \quad \phi \geq 0} \tag{24}$$

- ϕ: How important are old ideas in producing new ones?
- λ: How does the average productivity of research depends on the number of researchers around?
- We assume that individual researchers take $\overline{\theta}$ as given $\Rightarrow$ externalities. Are these positive or negative?

R&D sector

- When a new design is discovered, the inventor obtains a perpetual patent to produce the good. The inventor sells the patent to an intermediate-good producer and uses that money to consume and save
- What is the value, P_A, of a new design?

$$rP_A = \pi + \dot{P}_A. \tag{25}$$

- LHS: put P_A in the bank today and get rP_A as a return
- RHS: buy patent, get profits π and then sell the patent making a return $\dot{P}_A$
- Perfect capital markets $\Rightarrow$ LHS=RHS
- Rewriting (25): $r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A}$
- In the BGP $\frac{\dot{P}_A}{P_A} = n \Rightarrow P_A = \frac{\pi}{r-n}$
- Why? in BGP r is constant because K/Y is constant as in the Solow model. π in turn is proportional to Y/A, which means that $\dot{\pi}/\pi = \underbrace{\dot{Y}/Y}_{=g_a+n} - g_A$

Closing the model

- Arbitrage in labor markets: a worker must be indifferent between working in the final-good or R&D sector:

$$(1-\alpha)\frac{Y}{L_Y} = w_Y = w_R = \overline{\theta} P_A \tag{26}$$

  This determines the share of the population devoted to production and research respectively

- Profits associated with the production of intermediate goods provide the incentive for researchers to conduct R&D
- Capital is paid less than its marginal product and in general the competitive equilibrium is not Pareto-optimal
- In the BGP: $g_y = g_k = g_A$ following the same argument as in the Solow model with exogenous technological change
- Take logs and differentiate (24) w.r.t. t and note that the share of population devoted to research needs to grow at rate n in the BGP. This results in:

$$\boxed{g_A = \frac{\lambda n}{1-\phi}} \tag{27}$$

Schumpeterian growth

- In the Romer model, old intermediates are used forever: stone tools, arrows, Ptolemaic astronomy, steam engines,...
- In a model of vertical innovation, a new idea is always preferred to an old one — old ideas are continuously replaced by new ones
- Final good production: $Y = L_Y^{1-\alpha} A_i^{1-\alpha} X_i^{\alpha}$. At any point in time there is only one intermediate good in use instead of many. i indexes the state of technology so a higher number means better technology: $i > i' \Rightarrow A_i > A_{i'}$
- A_i jumps discretely over time as innovations occur and i increases
- The profit maximization problem for final good producers is:

$$\max_{L_Y, X_i} L_Y^{1-\alpha} A_i^{1-\alpha} X_i^{\alpha} - wL_Y - p_i X_i \tag{28}$$

- Firms hire labor and buy the intermediate good up to the point in which their marginal product equals the factor's price

Intermediate good producers

- There is a monopolist producing the intermediate good i
- Its profit maximization problem is:

$$\max_{X_i} \pi_i = \alpha L_Y^{1-\alpha} A_i^{1-\alpha} X_i^{\alpha} - r X_i \tag{29}$$

- Taking the FOC w.r.t. X_i yields: $X_i = \left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} L_Y A_i$
- $\Rightarrow p_i = \frac{r}{\alpha}$, i.e. the monopolist charges a constant markup $1/\alpha > 1$ above the marginal cost, r
- Plugging the optimal quantity of the intermediate produced into the profit function yields $\pi = \alpha(1-\alpha)Y$
- Since $X_i = K$, it follows that $Y = K^{\alpha}(A_i L_Y)^{1-\alpha}$

R&D Sector

- All researchers are working on the same idea — version $i+1$ of the capital good
- An individual doing research has a constant probability of discovering the new idea, $\overline{\mu}$. The size of the innovation improvement is assumed to be constant, i.e. $\frac{A_{i+1}}{A_i} = 1+\gamma$
- Again, we assume that $\overline{\mu} = \theta L_A^{\lambda-1} A_i^{\phi-1}$
- A successful researcher obtains a patent that lasts forever, but of course when version $i+2$ comes along nobody will want to buy version $i+1$ of the capital good and the patent is worthless ('drastic innovation')

- Arbitrage:

$$rP_A = \pi + \dot{P}_A - \overline{\mu} L_A P_A \tag{30}$$

Closing the model

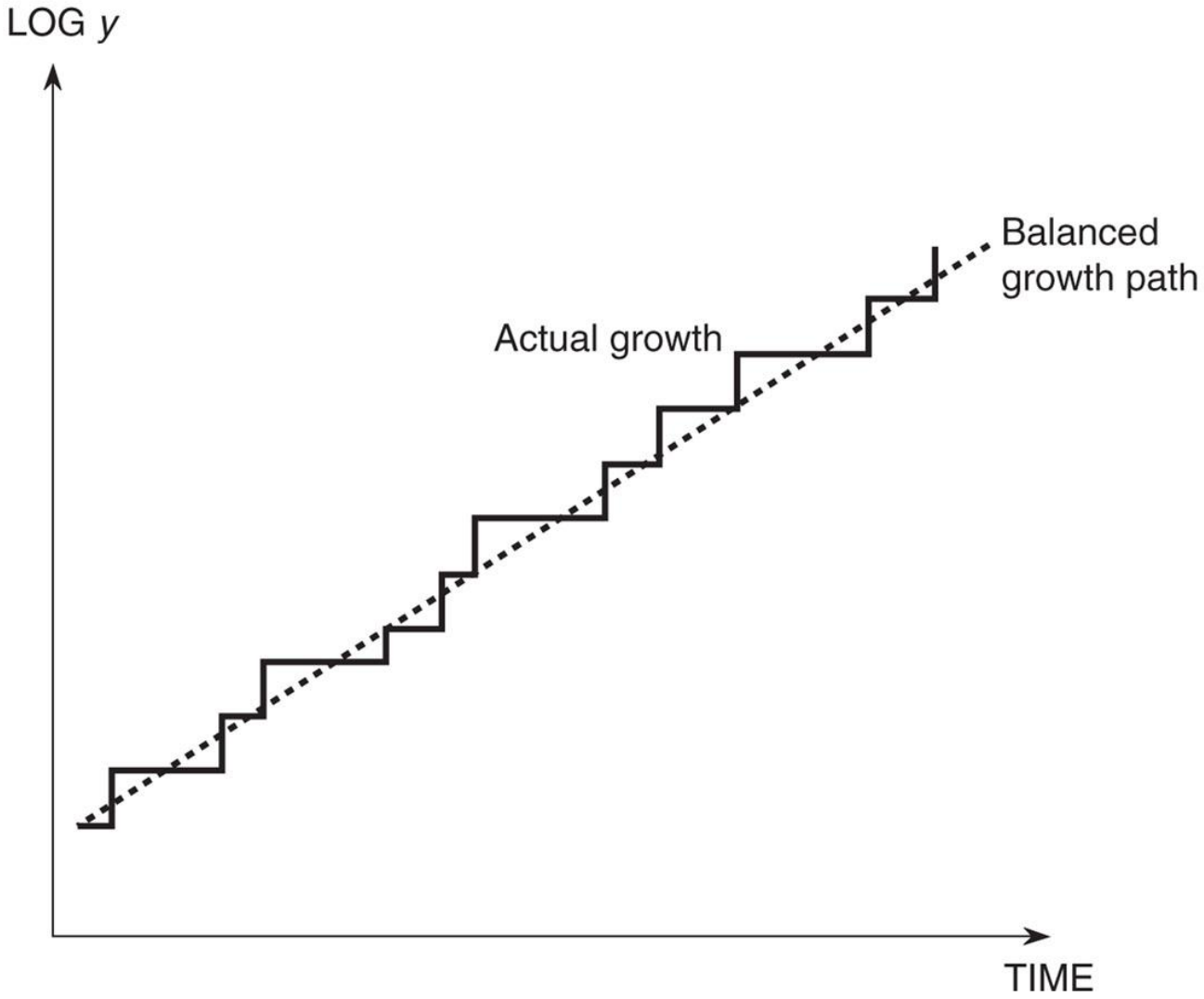
- Just as in the Romer model (with the exception of the last equality), we have that:

$$g_y = g_k = g_A = E\left[\frac{\dot{A}}{A}\right] = \frac{\lambda n}{1-\phi}. \tag{31}$$

  in BGP
- The value of a patent falls as the probability of innovation increases
- As the size of the innovation increases the value of a patent rises
- In both the Romer and Schumpeterian models, the growth rate of output per worker is pinned down by n
- However the Schumpeterian model has a richer market structure that is useful in understanding how competition shapes innovation
- The Romer setup is actually quite useful in an international trade context

Balanced growth path

FIGURE 5.4 **INCOME PER CAPITA ALONG BALANCED GROWTH PATH, SCHUMPETERIAN MODEL**



Discussion

- Notice that both models rely on a perfectly functioning intellectual property system to enforce patents
- A natural question to ask is: 'What is the optimal set of institutions that support the production and distribution of ideas?'
- This question has important implications to understand the great variation we observe in the level and growth rates of output per worker across countries
- Poor countries are poor not only because they have less physical and human capital per worker than rich countries, but also because they use their inputs much less efficiently
- "Bad" institutions can distort the adoption and utilization of ideas originating in developed countries. The follow-up question is also very interesting: 'Why are bad institutions so hard to get rid of?'

Schumpeter and the role of market structure

- The issue of what is the most effective organization to promote innovation is an extremely important issue in economic policy (and a key reason why Jean Tirole got the Nobel Prize in 2014)
- The incentive to innovate is the difference in profit that a firm can earn if it invests in R&D compared to what it would earn if it did not invest (recall the arbitrage condition)
- These incentives depend upon many factors including the characteristics of the invention, the strength of intellectual property protection, the extent of competition before and after innovation, barriers to entry in production and R&D, and the dynamics of R&D

Schumpeter and the role of market structure

- How good is the protection granted by the IP regime? tougher competition might dilute the value of conducting research and reduce innovation
- On the other hand, monopolists have less incentives to innovate because they lose the profits associated with the old technology they operated ('replacement effect')
- Or innovations might require large complementary investments that increase barriers to entry for new researchers
- Innovations might also produce differentiated products as in Romer $\Rightarrow$ this might facilitate a firm's ability to discriminate its consumers and earn higher profits
- Monopolists might have a greater incentive to innovate in order to preempt the entry of potential competitors to the market

Some criticism of endogenous growth models

- Their focus on the very long run and on incentives for expanding the technological frontier are not particularly useful for most developing countries, whose primary interest is in restoring short- to medium-term growth and accelerating technological catchup
- Thus, the question remains: 'what actions can be taken to spur more rapid economic growth over a relevant time horizon?'
- Perhaps we should aim at a more modest (but more useful goal): identify the binding constraints that forbid a country from moving to a 'better state' (e.g. from collapse to non-converging growth, or from poverty trap to rapid convergence)

Key questions:

- Describe the main assumptions of the Solow model
- Derive the law of motion of capital per worker in the Solow model without technical change
- Describe the transition dynamics of the Solow model when an economy is not in its steady state
- Explain why the equilibrium growth rate of output per worker in the Solow model without technical change is zero
- Is the Solow model able to reproduce the Kaldor growth facts?
- What is the difference between absolute and conditional convergence? Does conditional convergence imply absolute convergence? What about the other way around?
- Why are endogenous growth models important? What shortcomings of the Solow model are being addressed by these?
- Describe the main components of the Romer (1990) and Schumpeterian models and explain their main findings
- What are the main determinants of the growth rate of output per worker in the Romer model?
- Is the competitive equilibrium in the endogenous growth family of models Pareto optimal? If not, why?
- Contrast the implications derived from Romers endogenous growth model with those of the Solow model regarding the effect of economic policy on the growth rates of output per worker

References

- Jones, C. I. & D. Vollrath (2013) *Introduction to Economic Growth*. 3rd ed. Chapters 1-5
- Pritchett, L. (2006) "The Quest Continues", *Finance & Development*, Vol 43