Macroeconomics: Economic Cycles, Frictions and Policy

Economic Growth

Practice Problems

September 2019

1. **Introducing an income tax in the Solow model:** suppose that the government levies an income tax, τ, on both wage income and capital income. Instead of receiving $wL + rK = Y$, consumers receive $(1-\tau)wL + (1-\tau)rK = (1-\tau)Y$. For simplicity assume that all revenue collected by the government is used in wasteful activities. Trace the consequences of this tax for output per worker in the short and long-run, starting from steady state.

2. **Immigration in the Solow model:** assume that there is no productivity growth, that the population grows at a constant rate n, the savings rate is the constant $s \in (0,1)$, and the depreciation rate of physical capital is the constant $\delta \in (0,1)$. Suppose that the economy is initially in the steady state.

   (a) Now suppose that there is a one-time increase in the labor force from immigration, but n remains constant. Analyze the short-run and long-run effects of this change for the levels of wages and per-capita output, and the growth rates of total output and per-capita output.

   (b) Now suppose that immigration is a continuing process so that n increases to a higher value n'. Analyze the short-run and long-run effects of this change for the levels of wages and per-capita output, and the growth rates of total output and per-capita output.

3. **Introducing land in the Solow model:** Assume that there is a fixed amount of land T available in the economy for production, and that output is produced according to:

$$Y = AK^{\alpha}T^{\beta}L^{1-\alpha-\beta}, \quad \alpha + \beta < 1.$$

   There is exogenous technological progress, that is, $\frac{\dot{A}}{A} = g_A$ and exogenous population growth, $\frac{\dot{L}}{L} = n$. Capital accumulates according to the following law of motion:

$$\dot{K} = sY - \delta K,$$

   where $\dot{x}$ indicates the derivative of variable x with respect to time, $s \in (0,1)$ is the exogenous saving rate and $\delta \in (0,1)$ is the depreciation rate of physical capital.

   (a) Show that total output can be expressed as a function of the capital-output ratio,

$$Y = A^{\frac{\alpha}{1-\alpha}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} T^{\frac{\beta}{1-\alpha}} L^{1-\frac{\beta}{1-\alpha}}.$$

   This expression follows straightforwardly by diving both sides of the equation above by Y^{α} and rearranging the exponents.

(b) Using the expression you found in (a), find the growth rate of total output and the growth rate of output per worker along the balanced growth path (note that K/Y is constant along the balanced growth path —why?).

(c) Provide an economic intuition for the growth rate of output per worker. What happens to output per worker in the long run if $g_A = 0$?

4. **Endogeneizing the saving rate in the Solow model:** Consider the following version of the Solow model in discrete time. The economy has a constant population (i.e. $L_t = L \quad \forall t$) of a large number of identical agents that have preferences of the form:

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad \beta \in (0,1), \quad \text{and} \quad u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma \in [0, \infty). \tag{1}$$

The production technology is given by:

$$Y_t = K_t^{\alpha} \left(A_t L\right)^{1-\alpha}, \quad \alpha \in (0,1), \tag{2}$$

and there is exogenous technological progress, i.e. $A_t = (1+\mu)A_{t-1}$, with $\mu > 0$. Capital accumulates according to the usual law-of-motion:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{3}$$

where $\delta \in (0,1)$ is the depreciation rate of physical capital.

(a) Show that in the balanced growth path, it has to be the case that capital per unit of effective labor is constant.

(b) What is the balanced growth path rate of consumption growth?

(c) How is the steady-state level of $\frac{K}{AL}$ affected by the discount factor (β), the depreciation rate (δ) and the willingness of inter-temporal substitution (σ)? Provide an economic intuition.