Macroeconomics: Economic Cycles, Frictions and Policy

The Real Business Cycle Model

Practice Problems

September 2019

1. Consider the following non-stochastic version of the real business cycle model. The social planner's problem is:

$$\max_{C_t, K_{t+1}, L_t} \quad \mathcal{U} = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \tag{1}$$

subject to:

$$u(C, L) = \ln(C) + \phi \ln(L)$$

$$C_t + I_t = Y_t$$

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

$$K_{t+1} = (1 - \delta) K_t + I_t$$

$$L_t + N_t = 1$$

$$K_0 > 0 \quad \text{given.}$$

Where C_t, K_t, L_t, N_t denote respectively, consumption, physical capital, leisure and hours worked. $\beta \in (0, 1)$ is the discount factor, $\alpha \in (0, 1)$ is the share of income that accrues to capital, $\delta \in (0, 1)$ is the depreciation rate of physical capital and $\phi > 0$ determines the importance of leisure on the individual's utility relative to consumption.

(a) Rewrite problem (1) as an unconstrained maximization problem in which the social planner has to choose the optimal quantities of future capital, K_{t+1} and individuals' labor input, N_t.

(b) Solve for the first-order necessary conditions that characterize the solution to the unconstrained problem that you defined in part (a).

(c) Compare the solution you obtained in part (b) with the first-order necessary conditions you would obtain in a two-period model in which consumers have the same momentary utility (i.e. $u(C, L) = \ln(C) + \phi \ln(L)$ in each period) and they take as given their income stream $\{Y_1, Y_2\}$ and the real interest rate r. Are the solutions to these problems different?

2. **A Simple Calibration Exercise:** Assume that individuals live for one period and derive utility from consumption c and leisure ℓ. Individual preferences are given by $u(c, \ell) = \ln(c - \bar{c}) + \alpha\ell$, where $\bar{c} > 0$ is a subsistence level of consumption and $\alpha > 0$.[1] The individual has 1 unit of time which can be used to work in the market for a real wage w, or for leisure.

(a) Write down the individual's budget constraint.

(b) Solve for the optimal allocation of consumption, c, and hours worked, $1 - \ell$.

(c) In 1995, the typical worker in the U.S. spent about one-third of his available time working in the market. In contrast in 1830, the average worker spent 55 percent of his available time working (i.e. $(1 - \ell)_{1830} = 0.55$ and $(1 - \ell)_{1995} = 0.33$). On the other hand, the real wage in the U.S. in 1995 was 9.15 times the wage rate of 1830 (i.e. $w_{1830} = 1$ and $w_{1995} = 9.15$). Use the expression you found in part (b) for the fraction of time devoted to work

[1] Note that because the natural logarithm is only defined for strictly positive values, this utility function implies that consumers really need to satisfy their basic needs $\bar{c}$. Otherwise their level of utility would be $-\infty$.

for 1830 and 1995 to solve for $\bar{c}$ and α. What was the share of subsistence consumption in total consumption $\bar{c}/c$ in 1830 and 1995? Interpret your results.

3. Table 1 provides business cycle statistics for the U.S. economy for the period 1948:1-2016:2. Based on the information provided in the Table answer the following questions:

Figure 1: Business Cycle Statistics for the U.S. 1948:1-2016:2

Variable	Standard Deviation	Relative Standard Deviation	First-order Autocorrelation	Cross-correlation with GDP						
				t-3	t-2	t-1	t	t+1	t+2	t+3
GDP	0.0166	1.00	0.848	0.339	0.611	0.848	1.000	0.848	0.611	0.339
Wages	0.0094	0.57	0.683	-0.051	-0.026	0.051	0.138	0.171	0.172	0.159
GDP deflator	0.0092	0.55	0.908	0.102	0.063	-0.028	-0.149	-0.260	-0.363	-0.438
Investment	0.0753	4.53	0.804	0.123	0.421	0.691	0.874	0.780	0.603	0.381
Hours	0.0197	1.18	0.902	0.551	0.754	0.877	0.879	0.701	0.441	0.178
Output per worker	0.0110	0.66	0.717	-0.435	-0.237	0.052	0.399	0.470	0.503	0.458
Durable consumption	0.0449	2.70	0.692	-0.078	0.191	0.435	0.638	0.629	0.537	0.415
Non-durable consumption	0.0129	0.78	0.818	0.130	0.374	0.603	0.777	0.766	0.644	0.466
Unemployment rate	0.0083	0.50	0.893	-0.554	-0.774	-0.898	-0.881	-0.687	-0.416	-0.152

(a) Which variables have a cyclical component more volatile than GDP?

(b) What variable exhibits the highest degree of persistence over the business cycle? Which one the least?

(c) Which variables are procyclical and which ones countercyclical?

(d) Determine whether each variable is leading, lagging or coincident with GDP.