

Macroeconomics: Economic Cycles, Frictions and Policy

Two-period Model of Consumption and Saving

Practice Problems

September 2019

1. In the 2-period model of consumption and saving assume that period t utility is given by $u(c_t) = c_t - \frac{\alpha}{2}c_t^2$. As usual $\beta \in (0, 1)$ denotes the discount factor, and y_t and s_t are income and savings in period t respectively; r is the rate of return on savings. Assume that the consumer has no initial wealth, and that $\beta(1+r) = 1$. The objective of the consumer is to maximize total lifetime utility.
 - (a) Set up the consumer's optimization problem and derive the Euler equation.
 - (b) Derive an expression for c_1 in terms of y_1 , y_2 , and r . What is c_2 ? Is there consumption smoothing in this example? Does current consumption depend on the timing of current income? Explain.
 - (c) Now suppose that $y_1 = 40$, $y_2 = 60$, and $r = 0.05$. Based on your answer in (b) above calculate what c_1 and c_2 are equal to. Calculate what savings (or borrowing) is during period 1. Depict the optimal consumption choice in a graph (make sure to label all axis, endowment and consumption points).
2. In the 2-period model of consumption and saving, assume that individuals can borrow/lend at an interest rate $r > 0$. Lifetime consumer preferences are given by $\mathcal{U} = u(c_1) + \beta u(c_2)$, and the period utility functions are given by $u(c_t) = -e^{-c_t}$, for $t = 1, 2$.
 - (a) Does the utility function $u(c_t)$ satisfy the usual assumptions about utility functions? (i.e. is it increasing and concave in c_t)?
 - (b) Solve the consumer's optimization problem. Find an expression for the optimal levels of consumption in both periods, c_1 and c_2 as a function of β , r , y_1 and y_2 .
 - (c) How does the individual's saving in period 1, $s_1 = y_1 - c_1$ depend on his current income y_1 ?
3. Assume that an individual's lifetime preferences in the 2-period model of consumption are given by:

$$\mathcal{U} = \min\{\alpha c_1, c_2\}, \quad \alpha > 0.$$

As usual individuals can borrow/lend at an interest rate $r > 0$.

- (a) Solve the consumer's optimization problem. Find an expression for the optimal levels of consumption in both periods, c_1 and c_2 as a function of r , y_1 and y_2 .
- (b) Calculate $\frac{\partial s_1}{\partial r}$, i.e. whether this individual's saving in period 1 increases or decreases when the real interest rate increases. Show that the sign of this derivative depends on the value of the parameter α and provide an economic intuition for your finding.