The Real Business Cycle (RBC) Model

Macroeconomics: Economic Cycles, Frictions and Policy

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Outline

Introduction

- Business cycle research studies the causes and consequences of the recurrent expansions and contractions in aggregate economic activity
- The idea that economic fluctuations are caused primarily by real factors has gone in and out of fashion several times over the last century
- In the 1930s, Burns and Mitchell began to document the existence of a remarkable set of business cycle regularities over time and across countries
- Simultaneity of movement of economic variables over the cycle $\rightarrow$ predict expansions and contractions
- Interest in the business cycle waned after the publication of Keynes' General Theory
- Shift towards explaining the forces that determine the level of economic output at a point in time (conditional on the prior history of the economy)

Introduction

- RBC's main contribution to economics $\rightarrow$ growth and cyclical fluctuations can be studied using the same model: The Neoclassical Growth Model!
- From an analytical perspective, RBC models emphasized the use of stylized artificial economies for assessing those features of actual economies that are important for business cycles
- Short-term fluctuations arise from individuals' desire to inter-temporally substitute current and future consumption and intra-temporally substitute consumption and leisure as an optimal response to shocks in the economy's production possibility set

The stochastic neoclassical growth model

- **RBC model** $=$ **the Solow model** $+$ **endogenous saving** $+$ **labor supply decision** $+$ **stochastic productivity shocks**
- **Preferences:** There is a large number of infinitely-lived agents with expected utility

$$\mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} b^t u(C_t, L_t), \quad b > 0 \tag{1}$$

- The momentary utility function $u(C, L)$ is increasing in both arguments and strictly concave
- In order to have a steady-state, we need $u(C, L)$ to take the form:

$$u(C,L) = \begin{cases} \frac{[C^{1-\zeta} L^{\zeta}]^{1-\sigma}}{1-\sigma}, & \text{if } \ \sigma > 0 \ \text{ and } \ \sigma \neq 1 \\ (1-\zeta)\ln(C) + \zeta \ln(L), & \text{if } \ \sigma = 1 \end{cases} \tag{2}$$

  with $\zeta \in (0, 1)$

- **Time endowment:** $L_t + N_t = 1$

The stochastic neoclassical growth model

- **Technology:** Production requires capital and labor inputs:

$$Y_t = A_t F(K_t, X_t N_t) \tag{3}$$

- The production function F is continuous, twice-differentiable, concave and homogeneous of degree 1. Moreover, F satisfies the Inada conditions:

$$\lim_{K \to 0} F_K(K, N) \to \infty \tag{4}$$

$$\lim_{K \to \infty} F_K(K, N) = 0 \tag{5}$$

- Technical progress is labor-augmenting
- A is a stochastic productivity shock, and X_t is the deterministic component of productivity, $X_{t+1}/X_t = \gamma > 1$ (the same g_A as in the Solow model with exogenous technological change)
- **Resource constraint:** $C_t + I_t = Y_t$
- **Capital accumulation:** $K_{t+1} = (1 - \delta)K_t + I_t$, with $\delta \in [0, 1]$
- **Initial conditions:** A_0, K_0, $X_0 > 0$ given

Detrending the growth model

- Since we're interested in the fluctuations around the growth path, we detrend all variables (except L_t and N_t) by dividing by X_t
- For any variable Y_t, let $y_t \equiv Y_t / X_t$
- The detrended model looks as follows:

$$\max_{\{\{c_t\},\{k_{t+1}\},\{L_t\}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \right], \quad \beta \equiv b\gamma^{1-\sigma} \tag{6}$$

s.t.:

$$c_t + i_t = y_t \tag{7}$$

$$y_t = A_t F(k_t, N_t) \tag{8}$$

$$L_t + N_t = 1 \tag{9}$$

$$\gamma k_{t+1} = (1-\delta)k_t + i_t \tag{10}$$

- Without loss of generality, we assume $X_0 = 1 \Rightarrow X_t = \gamma^t$
- RBC models often omit growth all together or simply start with the transformed economy

Optimal capital accumulation

- Let's assume that $\gamma = 1$. Capital variables now denote aggregate variables and let $A_t = e^{z_t}$
- The Social Planner problem involves choosing allocations of capital, consumption and leisure that maximize the representative consumer's utility subject to the aggregate resource constraints

$$\max_{\{\{C_t\},\{K_{t+1}\},\{L_t\}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right] \text{ s.t.:},$$

$$C_t + I_t = Y_t \tag{11}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{12}$$

$$Y_t = e^{z_t} F(K_t, N_t) \tag{13}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad \varepsilon \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2) \tag{14}$$

$$L_t + N_t = 1 \tag{15}$$

$$K_0 > 0 \text{ given}$$

- Because there are no market failures in this model (no externalities, no public goods, no imperfect competition, no information failures) $\rightarrow$ First welfare theorem holds $\rightarrow$ $CE = PO$
- Notice that there are no prices involved in the Social Planner's problem

Social Planner's problem

- We can write the problem more succinctly as follows:

$$\max_{K_{t+1},N_t} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u\Big(e^{z_t} F(K_t, N_t) + (1-\delta)K_t - K_{t+1}, 1 - N_t \Big) \right] \tag{16}$$

- The recursive formulation of this problem is given by:

$$v(K,z) = \max_{K',N} \left\{ u\Big(e^{z} F(K,N) + (1-\delta)K - K', 1 - N \Big) + \beta \sum_{z'} P(z'|z) v(K',z') \right\} \tag{17}$$

- The FOCs and the envelope theorem yield:

$$u_C(C_t, 1-N_t) = \beta \mathbb{E}_t \Big[u_C(C_{t+1}, 1-N_{t+1}) \Big[e^{z_{t+1}} F_K(K_{t+1}, N_{t+1}) + 1 - \delta \Big] \Big] \tag{18}$$

$$u_L(C_t, 1-N_t) = u_C(C_t, 1-N_t) \Big[e^{z_t} F_N(K_t, N_t) \Big] \tag{19}$$

Steady state

- We set z equal to its unconditional mean of 0. In steady state: $C_t = C_{t+1} = C, \quad K_t = K_{t+1} = K, \quad N_t = N_{t+1} = N \quad \forall t$
- From the FOCs:

$$F_K(K,N) = (1/\beta) - 1 + \delta$$

$$F_K(K,N) = r + \delta \tag{20}$$

$$u_L(C, 1-N) = u_C(C, 1-N) F_N(K,N) \tag{21}$$

- Because $F(K,N)$ is homogeneous of degree 1, both $F_K(K,N)$ and $F_N(K,N)$ are homogeneous of degree 0 $\rightarrow F_K(K,N) = F_K(K/N, 1)$
- The capital-labor ratio of the economy is going to be pinned down by the parameters β and δ
- Since the capital-labor ratio is fixed, so will be the real wage $F_N(K/N, 1)$
- This in turn will determine the optimal trade-off between current consumption and leisure

Decentralized equilibrium

- Many times the conditions for the 1^{st} welfare theorem do not hold and the competitive equilibrium doesn't coincide with the allocation chosen by the SP
- Need to solve for the decentralized competitive equilibrium
- This means defining the problem of households and firms with their respective constraints, and having prices that will guarantee that all markets clear
- The problem of the firm is static: every period it rents capital and labor from households at prices r_t and w_t to produce a single consumption good (with price normalized to 1)
- The problem of the firm is:

$$\max_{K_t,N_t} \left\{ e^{z_t} F(K_t, N_t) - r_t K_t - w_t N_t \right\}, \quad \forall t \tag{FP}$$

- FOCs:

$$r_t = e^{z_t} F_K(K_t, N_t) \tag{22}$$
$$w_t = e^{z_t} F_N(K_t, N_t) \tag{23}$$

Household's problem

- The problem of households is:

$$\max_{\{\{c_t\},\{k_{t+1}\},\{l_t\}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right] \tag{HP}$$

s.t.:

$$c_t + i_t = w_t(z, K) n_t + r_t(z, K) k_t$$

$$k_{t+1} = (1-\delta) k_t + i_t$$

$$n_t + l_t = 1$$

$$K_{t+1} = (1-\delta) K_t + I_t$$

$$z' = \rho z + \varepsilon'$$

$$k_0 > 0 \text{ given}$$

- Notice that households need to form expectations about the evolution of the aggregate stock of capital K and the productivity shock z, because these two determine factor prices $w(z, K)$ and $r(z, K)$
- This is not particularly important in this context in which there is a representative individual, since individual and aggregate decisions are consistent by construction, but matters a lot in models with heterogeneous consumers or firms

Recursive competitive equilibrium

Definition (Recursive competitive equilibrium)

A recursive competitive equilibrium for this economy consists of a value function $v(z, k, K)$; a set of decision rules, $g_c(z, k, K)$, $g_n(z, k, K)$ and $g_k(z, k, K)$ for the household; a corresponding set of aggregate per capita decision rules $G_c(z, K)$, $G_n(z, K)$ and $G_k(z, K)$; and factor price functions, $w(z, K)$ and $r(z, K)$, such that these functions satisfy:

(i) the household problem (solves problem [HP])

(ii) that firms maximize profits (solves problem [FP])

(iii) the consistency of individual and aggregate decisions, that is, $g_c(z, k, K) = G_c(z, K)$, $g_n(z, k, K) = G_n(z, K)$ and $g_k(z, k, K) = G_k(z, K)$, for all (z, K); and

(iv) the aggregate resource constraint: $G_c(z, K) + G_k(z, K) = Y(z, K) + (1 - \delta)K$

Parametrizing the growth model

- Remember our initial question: does a model designed to be consistent with long-term economic growth produce the sort of fluctuations that we associate with the business cycle?
- We know the main ingredients of the model. Now we need to choose functional forms for technology and preferences and for the parameters of the model
- We want our model to display balanced growth (Kaldor facts # 1 and #2)
- Moreover, we want the shares of labor and physical capital in national income to be constant (Kaldor fact # 4). Cobb-Douglas technology:

$$Y_t = e^{z_t} K_t^{\alpha} N_t^{1-\alpha}, \tag{24}$$

where α is the share of output that is paid to capital owners if capital is paid its marginal product

Parametrizing the growth model

- Remember from the steady-state conditions:

$$F_K(K,N) = r + \delta,$$
$$\alpha\left(\frac{K}{N}\right)^{\alpha-1} = r + \delta$$
$$\frac{K}{N} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} \tag{25}$$

- Notice that the Cobb-Douglas production implies $K/Y = (K/N)^{1-\alpha}$ in steady state. Therefore:

$$\frac{K}{Y} = \left(\frac{\alpha}{r+\delta}\right) \tag{26}$$

just as Kaldor fact $\#$ 5!

- From the law of motion for capital: $K_{t+1} = (1-\delta)K_t + I_t$, in steady state:

$$\delta K = I$$
$$\Rightarrow \frac{I}{Y} = \delta\frac{K}{Y} \tag{27}$$
$$\Rightarrow \frac{C}{Y} = 1 - \frac{I}{Y} \tag{28}$$

Parametrizing the growth model

- Again, remember from the steady-state conditions:

$$r = (1/\beta) - 1 \tag{29}$$

- Given an average annual return on the SP-500 index of 6.5% ($0.065/4 = 0.0163$ quarterly) implies a discount factor $\beta = 1/(1+r) = 1/1.0163 = 0.984$
- Given that we know α and r, we can use the equation determining K/Y in steady state, to choose δ such that $K/Y \approx 3.3$

$$\delta = \frac{\alpha}{(K/Y)} - r = \frac{.36}{3.3} - 0.0163 = 0.0928 \tag{30}$$

- Productivity process: taking logs of the Cobb-Douglas production function

$$z_t = \ln(Y_t) - \alpha \ln(K_t) - (1-\alpha)\ln(N_t) \tag{31}$$

- If you regress z_t on a linear trend, you can compute γ (the deterministic component of productivity)
- Using the residuals from this regression, estimate ρ, the persistence of the productivity process, z_t, and the standard deviation of the productivity innovations σ_ε
- Using quarterly data for the US you obtain: $\rho = 0.979$ and $\sigma_\varepsilon = 0.007$

Parametrizing the growth model

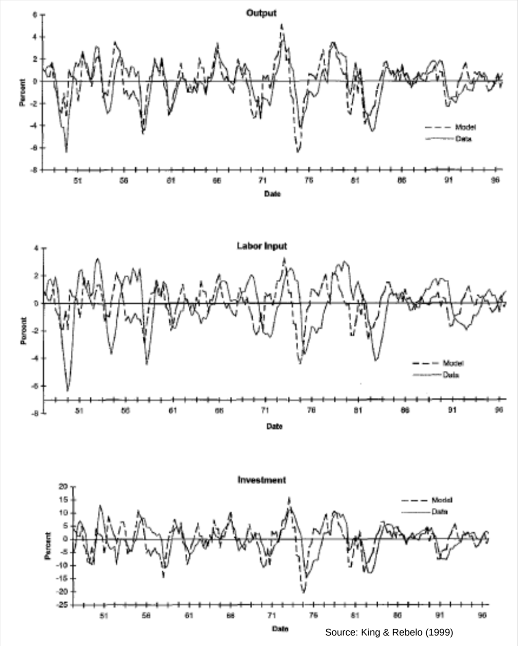
- For the postwar period per-capita hours worked in the marketplace remained roughly constant despite a continuous increase in the real wage. This suggest using preferences of the form:

$$u(c_t, l_t) = \frac{[c_t^{1-\zeta} l_t^{\zeta}]^{1-\sigma} - 1}{1-\sigma} \tag{32}$$

- However, non-market time $\neq$ leisure — very interesting secular changes on this margin
- A common parametrization for preferences is to assume that $\sigma = 1$, so

$$u(c, l) = (1-\zeta)\ln(c) + \zeta \ln(l) \tag{33}$$

Model vs. data



Source: King & Rebelo (1999)

RBC-generated moments

Table 3
Business cycle statistics for basic RBC model[a,b]

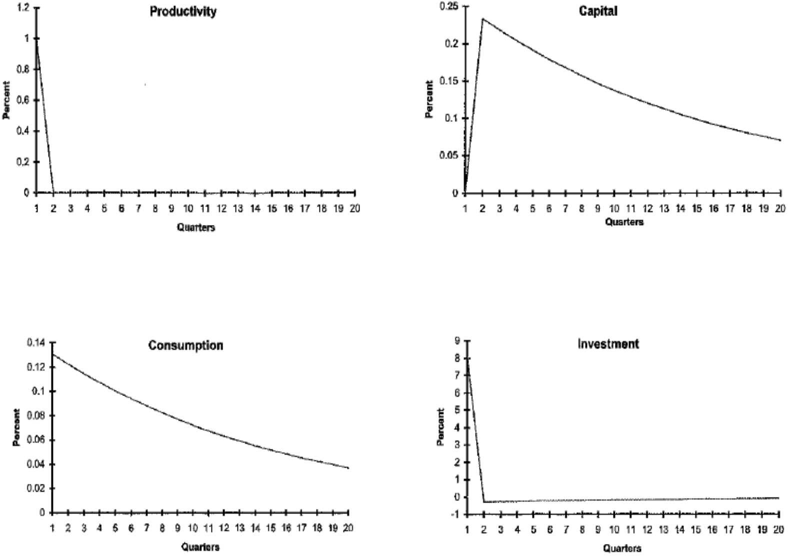
	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

[a] All variables have been logged (with the exception of the real interest rate) and detrended with the HP filter.

Source: King & Rebelo (1999)

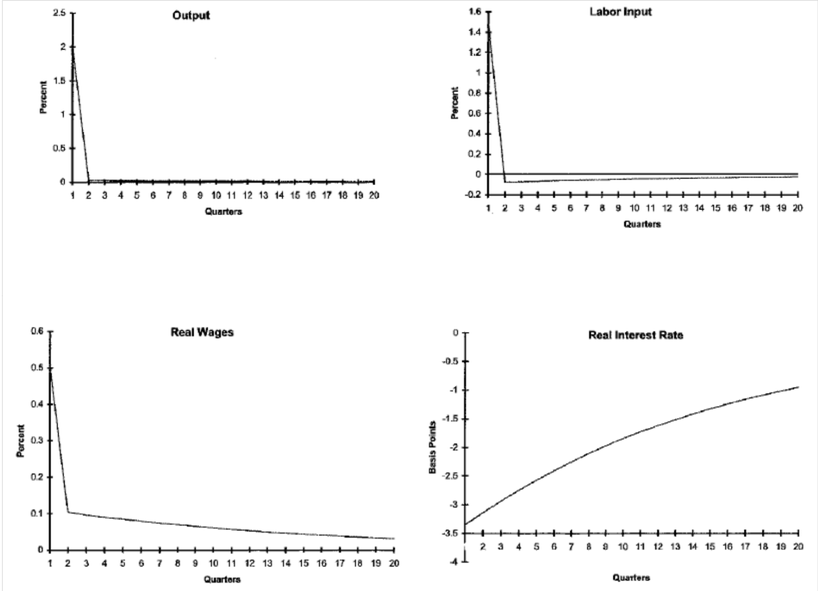
- $\sigma_{\text{model}}(Y) = 0.77\sigma_{\text{data}}(Y) \Rightarrow$ 1.39/1.81=0.77
- $\sigma_{\text{model}}(I)/\sigma_{\text{model}}(Y) = 2.93$ and $\sigma_{\text{data}}(I)/\sigma_{\text{data}}(Y) = 2.95$
- $\sigma_{\text{model}}(C_{nd})/\sigma_{\text{model}}(Y) = 0.44$ and $\sigma_{\text{data}}(C_{nd})/\sigma_{\text{data}}(Y) = 0.74$
- $\sigma_{\text{model}}(N)/\sigma_{\text{model}}(Y) = 0.48$ and $\sigma_{\text{data}}(N)/\sigma_{\text{data}}(Y) = 0.984$
- $\text{corr}_{\text{model}}(w, Y)$ is too high

Impulse-response analysis ($\rho = 0$)



Source: King & Rebelo (1999)

Impulse-response analysis ($\rho = 0$)



Source: King & Rebelo (1999)

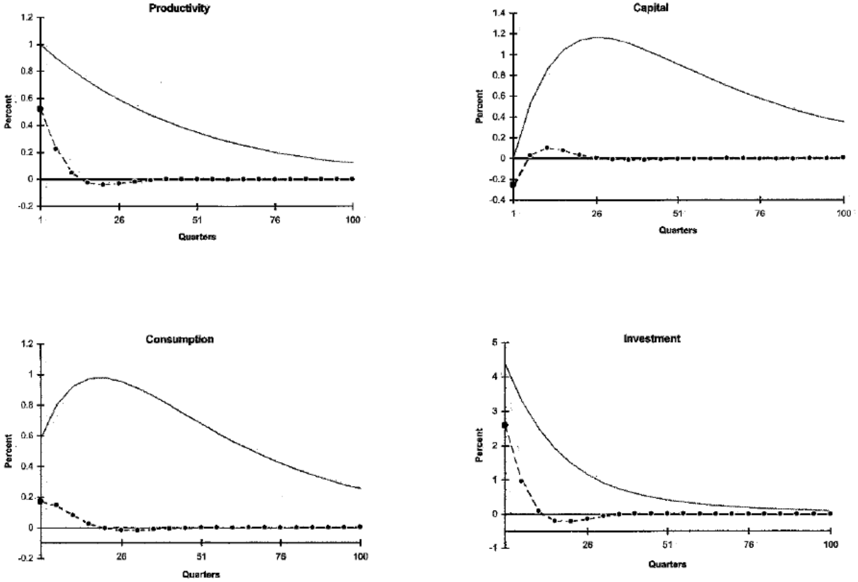
Impulse-response analysis

- Suppose that $\varepsilon_0 = 1$ and $\varepsilon_t = 0, \quad \forall t > 0$: The economy suffers a temporary positive productivity shock. What happens to the main aggregates in the economy?
- To fix ideas, let's assume that $\rho = 0 \Rightarrow$ this is a purely transitory productivity shock
- $W_0 = e^{z_0}(1-\alpha)Y_0/N_0 \uparrow \Rightarrow \quad N_0 \uparrow$
- The increase in N_0 amplifies the increase in Y_0 ($\uparrow 2\%$)
- What to do with this extra output? should individuals consume it all today?
- Consumers have a preference for consumption smoothing: they will increase C_0 but not 1-1 with output; the extra output is invested ($I_0 \uparrow$ by 8%)
- Main implication of the RBC model: the high volatility of investment doesn't arise because of "animal spirits" as suggested by Keynes. It is just the flip-side of consumption smoothing!
- What happens after period 0?
- The economy needs to shed the excess capital that it accumulated (gradually) by $\uparrow C$ and $\uparrow L$. The signal for consumers to do so is a lower real interest rate
- The propagation of shocks is very weak: we're unlikely to see several periods of high/low output

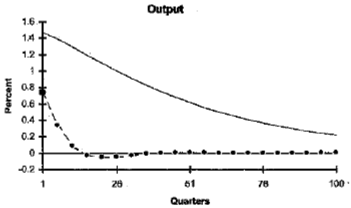
Impulse-response analysis

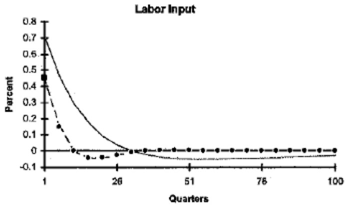
- Now, let's see what happens when we set $\rho = 0.979$ the value consistent with the observed Solow residuals for the US economy
- The same mechanisms are at work as before, but these effects are now drawn out over time
- Productivity is going to be above average for a extended period $\rightarrow \uparrow N$ and $\uparrow I$
- You can see that some time after the shock hits, the response of consumption, investment and labor looks very similar to the case of a transitory productivity shock
- The early part of the impulse responses is dominated by the fact that the productivity shock raises the desirability of work effort, production, investment and consumption
- The latter part is dominated by the reduction of capital back toward its stationary level
- With many periods of high output, there will be positive correlation between output and its past values: expansions and recessions will persist for many periods

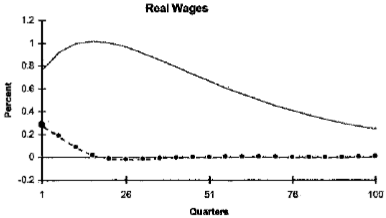
Impulse-response analysis ($\rho = 0.976$)

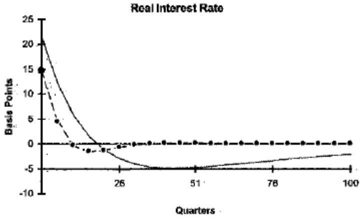


Impulse-response analysis ($\rho = 0.976$)

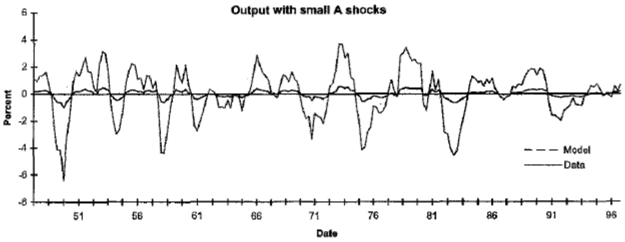


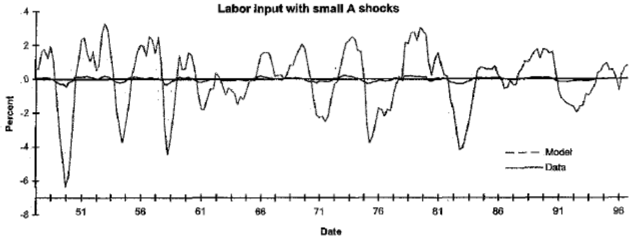






Simulated model with “small” productivity shocks





Source: King & Rebelo (1999)

Modern DSGE models

- The RBC model does not admit any role for changes in monetary policy to affect business cycle fluctuations
- For monetary policy to play a role $\rightarrow$ nominal rigidities = limits to price adjustments
- We also need firms that have some market power to set their own prices $\rightarrow$ monopolistic competition as in Romer (1990)
- Prices are set by multi-period contracts; each period some fraction of contracts expire and prices can be changed
- Price adjustment $\rightarrow$ New Keynesian Phillips curve:
  - Inflation depends on next period's expected inflation
  - The output gap
- Central bank operates according to a 'Taylor rule'. The nominal interest rate increases
  - More than one-for-one with inflation
  - When output is above its natural rate

Key questions:

- Explain what are the main assumptions underlying the benchmark RBC model
- Write explicitly the decision problem faced by individuals in the RBC model both in sequential and recursive form. What are the state variables? What variables does the individual choose?
- What is the mechanism in the RBC model that results in consumption being less volatile than output, and output in turn being less volatile than investment?
- Explain intuitively how consumption, investment, savings and work effort in this economy respond to a transitory improvement in total factor productivity

References

- King, R. and S. Rebelo. 1999. "Resuscitating Real Business Cycle Models", in John Taylor and Michael Woodford, eds., *Handbook of Macroeconomics* vol. 1A. Amsterdam: Elsevier. Chapter 14
- Prescott, E. 1986. "Theory Ahead of Business Cycle Measurement" Federal Reserve Bank of Minneapolis Quarterly Review