The Search & Matching Model of Unemployment

Macroeconomics: Economic Cycles, Frictions and Policy

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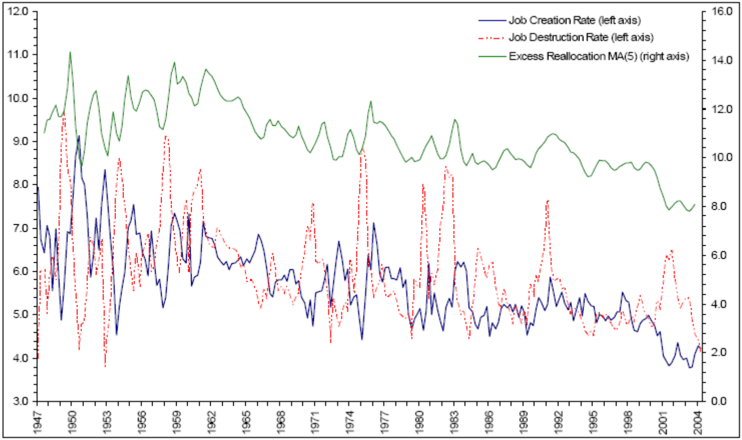
Outline

1. Motivation
2. The matching function
3. The Beveridge curve
4. Job creation
5. Wage curve
6. Steady-state equilibrium
7. Comparative statics

Job creation and job destruction

Figure 5. Quarterly Job Flows in Manufacturing, Seasonally Adjusted, 1947-2004



Source: Authors' tabulations and splicing of data from the MTD, LRD and BED. See text for details.

Source: Davis, Faberman and Haltiwanger (2006)

More than 10% of U.S. workers separate from their employers each quarter!

Motivation

- Why are so many people unemployed at the same time that there are many vacancies that go unfilled?
- Trade in the labor market is a decentralized activity
- It is uncoordinated, time-consuming and costly for both firms and workers
- Because both firms and workers have to spend resources before job creation and production can take place $\rightarrow$ jobs command rents in equilibrium
- This contrasts with the instantaneous adjustment that characterized the Walrasian labor market we have assumed so far

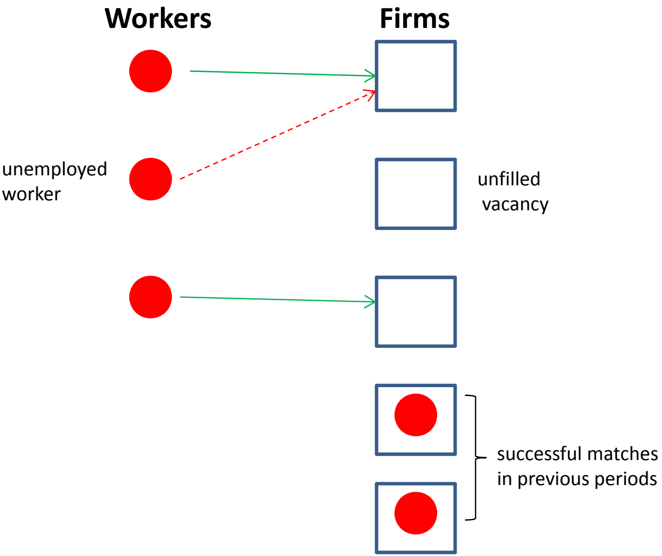
Main assumptions

- There is a measure L of infinitely-lived, risk-neutral individuals $\rightarrow u(c) = c$
- We abstract from the consumption/saving decision of individuals
- The discount factor for both workers and firms is $\beta = (1 + r)^{-1}$
- Their objective is therefore to maximize expected present discounted income
- Each firm employs at most 1 worker
- Each firm-worker match produces y units of output (sold at a price of 1)
- All unemployed workers search for jobs (no extensive margin of labor force)
- Only unemployed workers search for jobs (no on-the-job search)
- No differences in job search intensity (no search effort)
- Firm-worker matches end exogenously $\rightarrow$ neither firms nor workers have an incentive to terminate their relationship once they've been matched

Matching function

- We assume the existence of an aggregate matching function $\rightarrow$ number of jobs formed at any moment in time as a function of the number of workers looking for jobs and the number of firms looking for workers
- The matching function is an aggregate object. It summarizes the outcome of the investment of resources by firms and workers in the trading of labor resources as a function of the inputs
- The idea behind the matching function is to capture the fact that heterogeneity, frictions and information imperfections are pervasive in the labor market
- Think about an urn-ball problem: Firms are urns and workers are balls. Assume that all workers and firms are ex-ante identical
- If only one worker can occupy each job, an uncoordinated application process will lead to overcrowding in some jobs and no applications in others
- The friction here is the lack of information about other workers' actions

Matching function



Matching function

- Let u denote the unemployment rate (the fraction of unmatched workers) and v the vacancy rate (the number of vacancies as a fraction of the total labor force)
- The number of job matches occurring in the economy every period is given by:

$$mL = m(uL, vL) \tag{1}$$

- $m(u, v)$ is: (i) increasing in both arguments; (ii) strictly concave and (3) linearly homogeneous
- In every period, a fraction $m(uL, vL)/vL$ of vacancies are filled, and a fraction $m(uL, vL)/uL$ of unemployed workers find a job. Because m is linearly homogeneous, we can express these as follows:

$$\frac{m(uL, vL)}{vL} = \frac{Lm(u, v)}{vL} = \frac{m(u, v)}{v} = m(u/v, 1) \tag{2}$$

- Let $\theta \equiv v/u$, a measure of labor market 'tightness'. Thus:

$$\text{Prob. of filling a vacancy} = q(\theta) \tag{3}$$

$$\text{Prob. of finding a job} = \theta q(\theta) \tag{4}$$

Duration formula

- $\theta q(\theta)$ is the probability that that an unemployed worker finds a job (let me denote this probability by p)
- Suppose there's an unemployed individual in period t
- With probability p he has a 1 period unemployment spell
- With probability $(1-p)p$ he has a 2 period unemployment spell
- With probability $(1-p)^2p$ he has a 3 period unemployment spell, and so on...

$$\text{avg duration} \equiv Z = p \cdot 1 + p(1-p) \cdot 2 + p(1-p)^2 \cdot 3 + \cdots \quad (5)$$

$$Z = p[1 + 2(1-p) + 3(1-p)^2 + 4(1-p)^3 + \cdots] \quad (6)$$

$$Z(1-p) = p[(1-p) + 2(1-p)^2 + 3(1-p)^3 + 4(1-p)^4 + \cdots] \quad (7)$$

- Subtracting (7) from (6):

$$Z[1-(1-p)] = p[1 + (1-p) + (1-p)^2 + (1-p)^3 + \cdots]$$

$$Zp = \frac{p}{1-(1-p)}$$

$$Z = \frac{1}{p} \quad (8)$$

The Beveridge curve

- A match between a worker and a firm can ends due to an exogenous 'death shock' that arrives with probability λ. This is the only source of job destruction in this simplified model
- Firm-worker pairs experiencing the death shock are randomly selected $\rightarrow$ every period the number of workers moving into unemployment is:

$$\lambda(1-u)L \tag{9}$$

and the number of workers moving out of unemployment is:

$$\theta q(\theta)uL \tag{10}$$

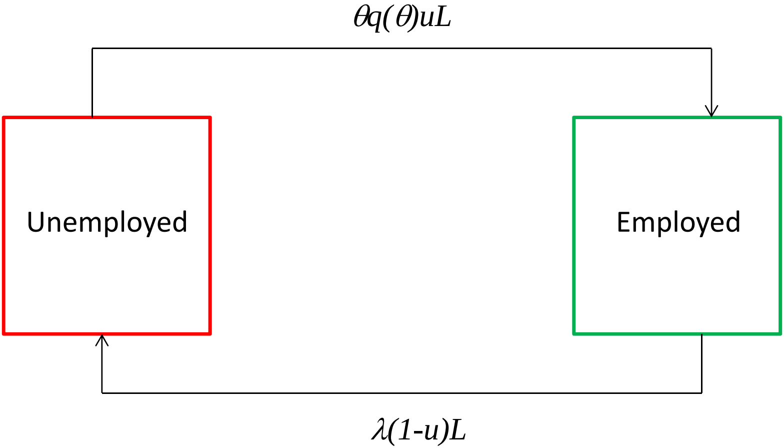
by the law of large numbers

- The law of motion of the unemployment rate u is: $\dot{u} = \lambda(1-u) - \theta q(\theta)u$
- In steady state when the flow of workers finding a job exactly matches the flow of workers experiencing termination shocks ($\dot{u}=0$) we have:

$$\boxed{\dot{u} = 0 \quad \Leftrightarrow \quad u = \frac{\lambda}{\lambda + \theta q(\theta)}} \tag{Beveridge Curve}$$

- This is a downward-sloping schedule (show this using the implicit function theorem). Higher θ increases the probability of finding a job

Employment flows



Job creation

- An unmatched firm (wanting to hire a worker) pays a fixed cost c per period to post a vacancy
- There are two possible states for a firm: (i) occupied job or (ii) open vacancy
- Let J denote the expected PDV from an occupied job and V the expected PDV of a vacancy. Two Bellman equations are satisfied in steady state:

$$J = y - w + \beta\Big[\lambda V + (1-\lambda)J\Big] \tag{11}$$

$$V = -c + \beta\Big[q(\theta)J + (1-q(\theta))V\Big] \tag{12}$$

- We assumed that the size of the economy is L, but we haven't said how many firms are operating there
- Free entry $\rightarrow$ firms will enter the market up to the point in which the value of an open vacancy, V, is equal to 0
- Using this assumption in equation (12) $\rightarrow$ $J = \frac{c}{\beta q(\theta)}$

Job creation

- Substituting the value of J in equation (11) and using the fact that $\beta = (1+r)^{-1}$:

$$\boxed{\frac{y-w}{r+\lambda} = \frac{c}{q(\theta)}} \tag{JC curve}$$

- $(y-w)/(r+\lambda)$ is the marginal benefit of hiring a worker, discounted using the interest rate, r, and taking into account that at any moment there's a probability λ that the match will end
- $1/q(\theta)$ is the expected amount of time it takes to fill a vacancy $\rightarrow$ expected cost of having a vacancy open is $c/q(\theta)$
- Rewrite the Job creation curve as:

$$w = y - \frac{r+\lambda}{q(\theta)}c \tag{13}$$

- Remember that $q'(\theta) < 0$

$$\theta \uparrow \Rightarrow q(\theta) \downarrow \Rightarrow \frac{r+\lambda}{q(\theta)}c \uparrow \Rightarrow w \downarrow \tag{14}$$

- Intuitively, if $w \downarrow \; \Rightarrow$ firms open more vacancies $\Rightarrow \quad \theta \equiv v/u \quad \uparrow$

Workers

- A worker earns w when employed, and searches for a job when unemployed, obtaining a flow income z (value of leisure/unemployment benefit/production in the informal sector)
- Similarly to the case of the firms, a worker can be in two states: (i) employed or (ii) unemployed
- The expected pdv of employed and unemployed workers are given by:

$$E = w + \beta\Big[\lambda U + (1-\lambda)E\Big] \tag{15}$$

$$U = z + \beta\Big[\theta q(\theta)E + (1-\theta q(\theta))U\Big] \tag{16}$$

- rU can be interpreted as the expected return on the worker's human capital during search $\rightarrow$ the minimum compensation required to search for a job
- Workers stay on their jobs as long as $E \geq U$
- A sufficient condition for this inequality to hold is $y > z$

Wage determination

- A match between a firm and worker yields a total return $> V + U$ $\rightarrow$ economic rent
- Note that V is the outside option for the firm, and U is the outside option for the worker
- This rent is split between the firm and the worker by means of a Nash bargaining solution. Letting $\phi \in [0, 1]$ denote the bargaining power of the worker and $S = (J - V) + (E - U)$ denote the total match surplus
- The match surplus is shared according to the following equation:

$$\max_{(E-U),J} (E - U)^{\phi} J^{1-\phi} \tag{17}$$

s.t.:

$$E + J - U = S \tag{18}$$

- The solution of this problem yields:

$$E - U = \phi S \tag{19}$$

$$J = (1 - \phi)S \tag{20}$$

- The worker receives a share ϕ of the total surplus. The remaining part goes to the firm

A lot of algebra...

- From equation (15) solve for E (use the fact that $\beta = (1+r)^{-1}$):

$$E = w + \beta[\lambda U + (1-\lambda)E] \tag{21}$$

$$\rightarrow E = \frac{1+r}{r+\lambda}[w + \beta\lambda U] \tag{22}$$

- This implies that $E - U$ is (again, use the fact that $\beta = (1+r)^{-1}$):

$$E - U = \left[\frac{1+r}{r+\lambda}\right]w - \left[\frac{r}{r+\lambda}\right]U \tag{23}$$

- From equation (11) solve for J (use the fact that $V = 0$):

$$J = y - w + \beta[\lambda V + (1-\lambda)J] \tag{24}$$

$$\rightarrow J = \frac{1+r}{r+\lambda}[y - w] \tag{25}$$

- Substitute these into the surplus equation $S = (E-U) + J$:

$$S = \frac{1}{r+\lambda}\left[y - rU\right] \tag{26}$$

- Finally, use the surplus splitting rule $(E-U) = \phi S$:

$$\left[\frac{1+r}{r+\lambda}\right]w - \left[\frac{r}{r+\lambda}\right]U = \frac{\phi}{r+\lambda}\left[y - rU\right] \tag{27}$$

A lot of algebra...

- Solving for w we have:

$$w = \frac{r}{1+r}U + \frac{\phi}{1+r}\Big(y - rU\Big) \tag{28}$$

- We can now solve for $rU/(1+r)$. Start from the Bellman equation for unemployed workers, equation (16)

$$U = z + \beta[\theta q(\theta)E + (1 - \theta q(\theta))U] \tag{29}$$

rearrange

$$(1-\beta)U = z + \beta\theta q(\theta)(E - U) \tag{30}$$

use $\beta = (1+r)^{-1}$

$$\frac{r}{1+r}U = z + \frac{\theta q(\theta)}{1+r}(E - U) \tag{31}$$

$$\tag{32}$$

- From the Nash bargaining solution you have that $E - U = \phi S$ and $J = (1-\phi)S$. This implies that,

$$E - U = \frac{\phi}{1-\phi}J \tag{33}$$

- And from the top page of slide # 12 (again, replacing $\beta = (1+r)^{-1}$), we have,

$$E - U = \frac{\phi}{1-\phi}\frac{(1+r)c}{q(\theta)} \tag{34}$$

- Replacing in equation (31):

$$\frac{r}{1+r}U = z + \frac{\phi}{1-\phi}\theta c \tag{35}$$

A lot of algebra...

- Finally, replacing equation (35) into equation (28) we have:

$$w = z + \phi(y - z + \theta c) \tag{36}$$

- We call this the wage curve equation

Wage curve

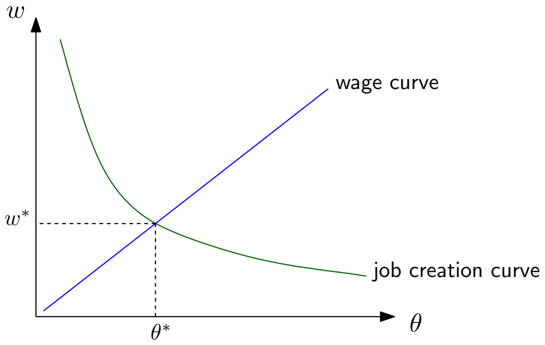
- After all this algebra we end up with an expression for the wage that looks like this:

$$\boxed{w = z + \phi(y - z + \theta c)} \qquad \text{(WC)}$$

- It means that the worker receives his outside option z (unemployment benefits, for instance) plus a fraction ϕ of the firm's output in excess of z and the average hiring cost per unemployed worker $c\theta = \frac{cv}{u}$
- Workers are rewarded for the saving of hiring costs that the firm enjoys when a job is formed
- Notice that the wage curve is upward sloping in the $\{w, \theta\}$ space. Higher θ means that the probability of leaving unemployment, $\theta q(\theta)$, is higher, which in turn means that the bargaining power of the worker is higher $\rightarrow$ higher w

Steady-state equilibrium

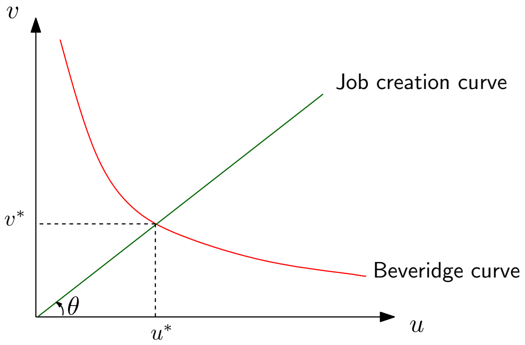
- A steady-state equilibrium is a triple $\{u, \theta, w\}$ that satisfies: (1) the Beveridge curve, (2) the job creation condition and the (3) wage curve, given an interest rate, r
- Putting together the WC and the JC curve in $\{w, \theta\}$-space we get:



- Equilibrium θ is independent of the level of unemployment

Steady-state equilibrium

- Substituting the WC into the JC curve:



Comparative statics: increase in y

- For reference, these are the 3 main equations of the model:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

$$w = y - \frac{r + \lambda}{q(\theta)} c$$

$$w = z + \phi(y - z + \theta c)$$

- Suppose that firms become more productive $y \uparrow$ (or that we're on business cycle boom). What happens?
  - $y \uparrow \Rightarrow$ JC shifts up and to the right (firms want to open more vacancies)
  - Notice that the WC also shifts up, but less so than JC (because $\phi \in (0, 1)$)
  - Both w and $\theta \uparrow$
  - In the $v - u$ space, a higher θ rotates the JC curve counterclockwise $\curvearrowleft$
  - $\Rightarrow v \uparrow$ and $u \downarrow$

Comparative statics: increase in z

- For reference, these are the 3 main equations of the model:

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}$$

$$w = y - \frac{r + \lambda}{q(\theta)} c$$

$$w = z + \phi(y - z + \theta c)$$

- Suppose that unemployment benefits $z \uparrow$ What happens?
    - $z \uparrow \Rightarrow$ WC shifts up
    - $w \uparrow$ and $\theta \downarrow$
    - JC is unaffected (z does not appear in the JC equation)
    - In the $v - u$ space, a lower θ rotates the JC curve clockwise $\curvearrowright$
    - $\Rightarrow v \downarrow$ and $u \uparrow$

Key questions:

- What is the mechanism that generates unemployment in the search & matching model?
- What is the Beveridge curve? Why do we need it in the search & matching model?
- How are wages determined in the search & matching model?
- What happens to equilibrium wages, unemployment and the vacancies/unemployment ratio if the government decides to increase unemployment benefits?
- What happens to equilibrium wages, unemployment and the vacancies/unemployment ratio if the productivity of firms decreases?

References

- Pissarides, C. A. 2000. *Equilibrium Unemployment Theory*, 2nd ed. MIT Press, Chapter 1.