A One-Period Model of the Macroeconomy

Macroeconomics: Economic Cycles, Frictions and Policy

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What is a macroeconomic model?

- A macroeconomic model is an artificial society characterized by mathematical representations of all participants' objectives, resources and interactions
- Participants: households, firms, government, central bank
- Agents in the model interact with each other through explicitly defined trading arrangements
- An equilibrium concept is specified so as to produce predictions about the interaction of the participants in the model
- Macroeconomics is the study of supply and demand in multiple markets at a time
- What happens and what is expected to happen in one market will affect what happens in the other markets under consideration

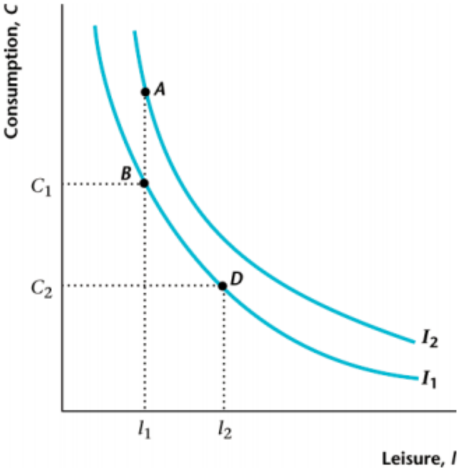
Discussion

- Why equilibrium? This is a situation that has a chance of persisting long enough $\rightarrow$ "good" predictions about the quantities and prices at which agents trade with each other
- Why to create an artificial society populated with *homo-oeconomicus*? We are looking for conditions under which the data originating from the real world emerges as the outcome of the interaction between self-interested agents
- We rely on models (that themselves are built upon a body of assumptions) to produce a compelling story of cause and effect. If someone doesn't like your assumptions they can change them and see whether they obtain different conclusions
- Macroeconomists cannot conduct controlled experiments
- Why math? Best way to keep models internally consistent
- Models allow us to answer normative questions, e.g. should marginal tax rates be lowered? Are trade deficits bad? Should the length of unemployment support be extended during recessions?

Consumer's preferences

- 'Representative' consumer whose actions mirror the average behavior of a collection of households
- This consumer makes decisions about how much to consume and how much time to devote to leisure
- The preferences of this representative consumer are represented by a utility function:
$$U(C, l) \tag{1}$$
where C denotes the quantity of consumption and l time spent outside work
- $U(C, l)$ allows the consumer to compare any combination of consumption and leisure
- Preferences are assumed to have the following properties:
  - More is always preferred to less: $U_C \equiv \mathrm{d}U/\mathrm{d}C > 0$ and $U_l \equiv \mathrm{d}U/\mathrm{d}l > 0$
  - Decreasing marginal utility of consumption and leisure: $U_{CC} \equiv \mathrm{d}^2U/\mathrm{d}C^2 < 0$ and $U_{ll} \equiv \mathrm{d}^2U/\mathrm{d}l^2 < 0$

Indifference curves

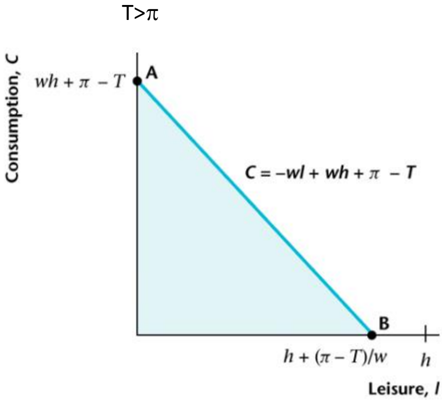


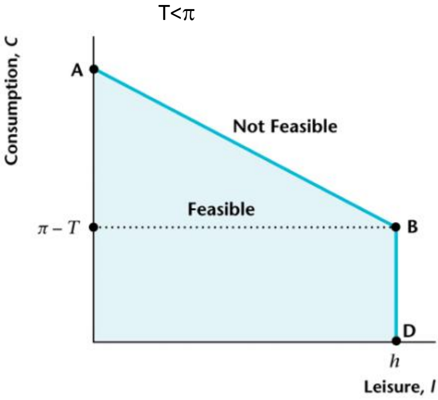
- $U(C_1, l_1) = U(C_2, l_2)$ since they are in the same indifference curve.
- $U(A) > U(B) = U(D)$. Why?

Budget constraint

- The consumer is a price-taker in both the goods and labor markets
- A consumer has an endowment of h hours that she can devote to work or to enjoy leisure:
$$l + n = h \tag{2}$$
- The consumer earns income by working, from the profits of a portfolio of firms she owns (π) and pays (lump-sum) taxes (T)
- Thus, the consumer's budget constraint is given by:
$$C = w(h - l) + \pi - T \tag{3}$$
where w denotes the prevailing market wage
- Note that we have already substituted the time constraint in the budget constraint above

Budget constraint





Consumer's utility maximization problem

- Maximize utility subject to budget constraint
- Set up Lagrangian:

$$\mathcal{L} = U(C, l) - \lambda \cdot (C - w(h - l) + \pi - T) \tag{4}$$

- First-order conditions (FOC) for an interior solution:

$$[C]: \ U_C(C, l) - \lambda = 0 \tag{5}$$

$$[l]: \ U_l(C, l) - \lambda w = 0 \tag{6}$$

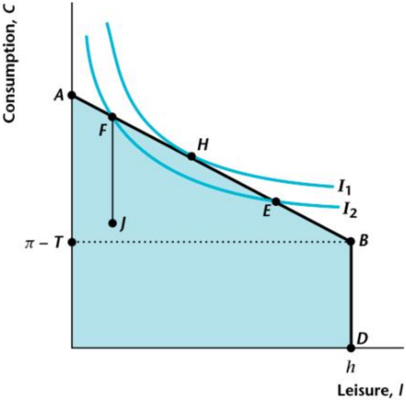
- Which implies:

$$\frac{U_l(C, l)}{U_C(C, l)} \equiv \mathrm{MRS}_{l,C} = \frac{w}{\underbrace{1}_{=p_C}} = w \tag{7}$$

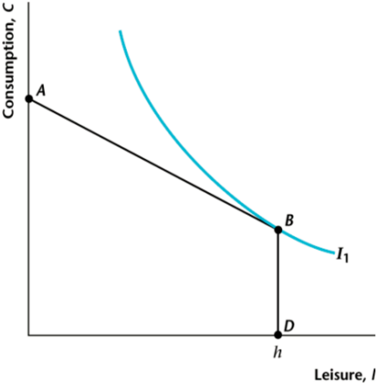
- Optimality: the rate at which a consumer is willing to substitute consumption for leisure (to keep utility constant) equals the relative price of leisure vis-à-vis consumption

Consumer's utility maximization problem

Interior solution



Corner solution



- The first panel shows an interior solution: $l^* \in (0, h)$
- When $l^* = h$ we have are in what's known as a 'corner' solution

Firms

- The representative firm owns capital (plant, machinery and equipment) (K) and hires workers (N) at the prevailing wage w to produce the consumption good
- A firm is represented by its technology, i.e. a recipe that transforms inputs into outputs:
$$Y = zF(K, N), \tag{8}$$
where Y denotes output of consumption goods, and z is total factor productivity (TFP), how much output is a firm able to produce with one bundle of inputs
- $F(K, N)$ is assumed to have the following properties:
    - More inputs $\Rightarrow$ more output: $F_K > 0$ and $F_N > 0$
    - Constant returns to scale: $F(\lambda K, \lambda N) = \lambda F(K, N)$
    - Decreasing marginal product for all factors: $F_{KK} < 0$ and $F_{NN} < 0$
- In our 1-period model, we assume that a firm's capital stock is fixed. The profit maximization problem faced by a firm is:
$$\max_N \pi = zF(K, N) - wN \tag{9}$$
Note that the price of the firm's output is set equal to 1

Optimal labor demand

- Firms choose how many workers to hire taking the prevailing wage w as given
- The solution to the firm's profit maximization problem (for a given capital stock K) is given by:
$$[N] : zF_N(K, N) = w \tag{10}$$
- Firms hire workers up to the point in which the marginal product of labor equals the wage
- Labor demand is downward-sloping. Taking the total differential of the equation above:
$$\frac{\mathrm{d}N}{\mathrm{d}w} = \frac{1}{zF_{NN}(K, N)} < 0 \tag{11}$$
- Similarly, higher TFP increases labor demand at any given wage:

$$\begin{aligned} \mathrm{d}z + F_{NN}(K, N)\mathrm{d}N &= 0 \\ \Rightarrow \frac{\mathrm{d}N}{\mathrm{d}z} &= -\frac{1}{F_{NN}(K, N)} > 0 \end{aligned} \tag{12}$$

Model parametrization

- So far, we have been very general about the specific functional forms that individual preferences and production technology take
- Some frequently used examples include:
- Preferences:
$$U(C,l) = \frac{C^{1-\sigma}}{1-\sigma} + B\frac{l^{\psi}}{\psi}, \quad \sigma, \psi, B > 0 \tag{13}$$
- Technology:
$$F(K,N) = zK^{\alpha}N^{1-\alpha}, \quad \alpha \in (0,1) \tag{14}$$
- The objects $\{\sigma, B, \psi, \alpha\}$ are called structural or 'deep' parameters
- The underlying assumption is that these structural parameters are not affected by economic policy — although individual and firm behavior changes in response to economic policy
- Calibration/estimation: The behavior of the agents in the artificial economy replicates salient features of 'real world' data
- What are the relevant features that we want our models to replicate?

Government

- Governments supply many different goods and services: infrastructure, education, health, defense
- Several of these goods & services are public goods that are (to different degrees) non-excludable and non-rival
- For the time being, we assume that the government just consumes a certain quantity of the consumption good
- Government expenditure is an exogenous variable in the model
- Furthermore, we assume that the government runs a balanced budget: $G = T$
- Notice that changes to government expenditure will affect individuals' labor supply choices; these in turn will have an effect on wages which in turn will affect output and employment...

Competitive equilibrium

- Exogenous variables: determined outside the model. In our simple model, these are G, K and z
- Endogenous variables: determined by the optimal choices of participants in the model. Namely, C, l, n, N, T, Y, π

Definition (Competitive equilibrium)

A competitive equilibrium is a set of allocations $\{C^\star, l^\star, Y^\star, T^\star\}$ and a vector of prices $\{w^\star\}$ such that given the exogenous variables G, K and z:

(i) The representative consumer chooses $C^\star$ and $l^\star$ so as to maximize her utility given her budget constraint

(ii) The representative firm chooses labor demand $N^\star$ so as to maximize profits, taking as given its technology

(iii) The labor market clears, i.e. $N^\star = h - l^\star$

(iv) The government budget constraint is satisfied, i.e. $G = T^\star$

Walras' law

- Notice that the definition of competitive equilibrium only states that the labor market needs to clear. What happens with the goods market?
- Walras' law: if all markets but one are in equilibrium, then that last market must also be in equilibrium
- When the labor market is in equilibrium $\Rightarrow$ the goods market is also in equilibrium

$$\begin{gathered} C^\star = w^\star N^\star + \pi - T \\ C^\star = w^\star N^\star + Y^\star - w^\star N^\star - G \\ \Rightarrow C^\star + G = Y^\star \end{gathered} \tag{15}$$

Production possibilities frontier (PPF)

- All combinations of (C, l) that are feasible to produce given the economy's resources and technology

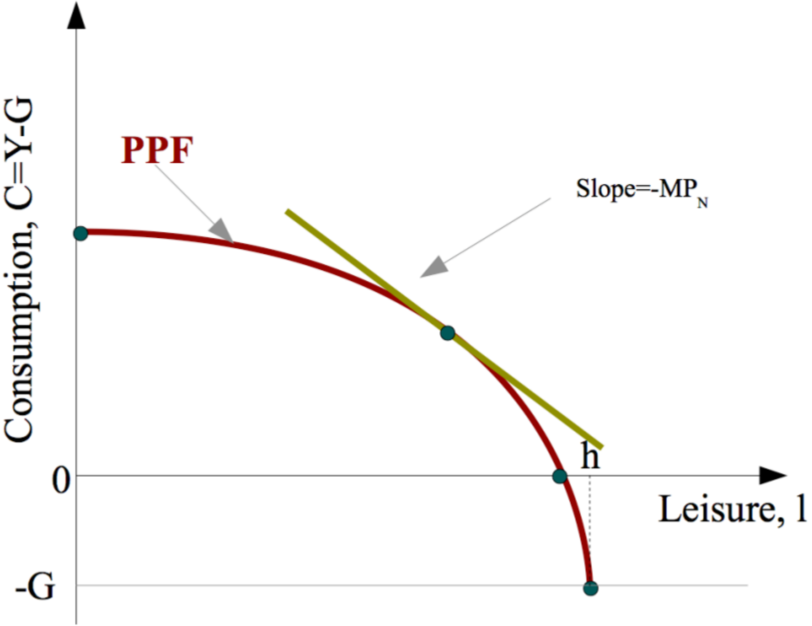
$$Y = zF(K, N)$$
$$C + G = zF(K, h - l) \tag{16}$$

- Note that:

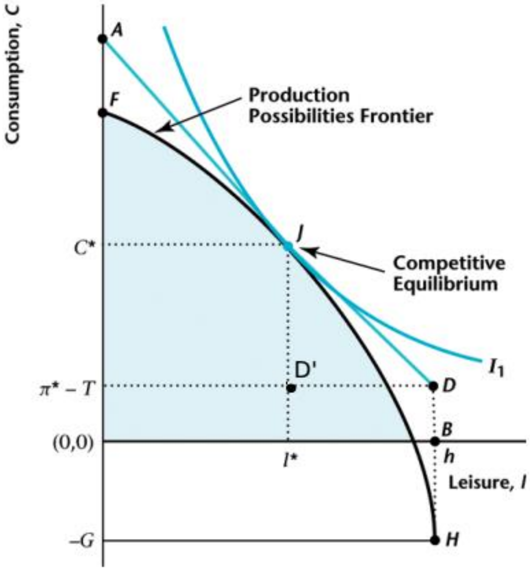
$$\frac{\mathrm{d}C}{\mathrm{d}l} = -zF_N(K, h - l) < 0 \tag{17}$$
$$\frac{\mathrm{d}^2C}{\mathrm{d}l^2} = zF_{NN}(K, h - l) < 0 \tag{18}$$

- PPF is downward-sloping and concave
- It's slope (in absolute value) is equal to the marginal product of labor and is called the marginal rate of transformation $\mathrm{MRT}_{l,C}$
- Also, $C(h) = -G$

Production possibilities frontier (PPF)



Competitive equilibrium



$$\text{MRS}_{l,C} = w = \text{MRT}_{l,C} \tag{19}$$

Pareto optimality

- Imagine there is a benevolent social planner that chooses the allocation of resources so as to maximize the utility of the representative consumer subject to the technology and resources constraints
- The social planner solves the following problem:

$$\max_{C,l} U(C,l) \tag{20}$$

$$\text{s.t.:}$$

$$C + G = Y = zF(K, h-l)$$

- The Lagrangian associated with this problem is:

$$\mathcal{L} = U(C,l) - \lambda \cdot (C + G - zF(K, h-l)) \tag{21}$$

- The FOC are:

$$[C] : \;\; U_C(C,l) = \lambda \tag{22}$$

$$[l] : \;\; U_l(C,l) = \lambda z F_N(K, h-l) \tag{23}$$

- Thus:

$$U_l(C,l)/U_C(C,l) = zF_N(K, h-l)$$

$$\text{MRS}_{l,C} = \text{MRT}_{l,C} \tag{24}$$

- Notice that there are no prices involved!

The fundamental welfare theorems

- Notice that the allocation of resources achieved under competitive equilibrium (CE) is exactly the same as the one that the social planner would have chosen
- There is no way to change the CE allocation to make someone in the economy better off while keeping everyone else at least as well off

Definition (1stfundamental theorem of welfare economics)

Under the assumption that both individuals and firms are price-takers in goods and factor markets, then a competitive equilibrium allocation is always Pareto optimal

- Despite individuals' ignorance or even complete disregard for the well-being of others, the price mechanism leads to an outcome in which no one can be made better off without making someone else worse off

Definition (2nd fundamental theorem of welfare economics)

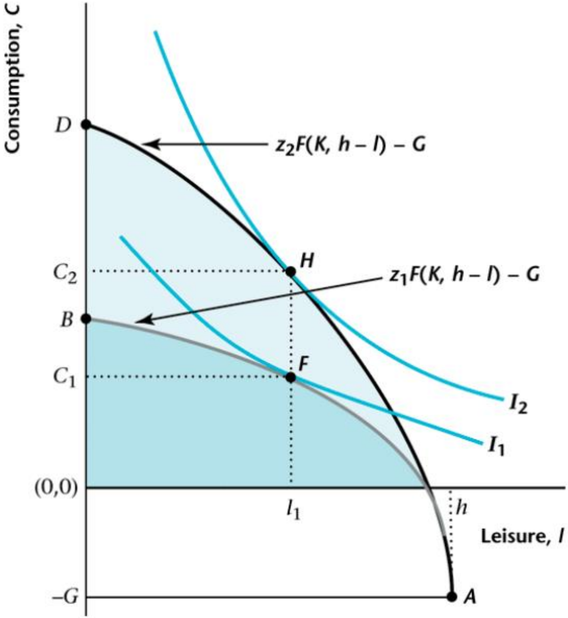
Every Pareto efficient allocation of an economy with convex preferences and technology is an equilibrium for a suitable price vector, given an initial redistribution of endowments

- We can find prices such that any Pareto efficient allocation can be a competitive equilibrium

Increase in TFP

- Comparative statics: what happens to the endogenous variables in the model when one of the exogenous variables (TFP in this case) changes?
- Since $Y = zF(K, N) = C + G$ we can see that consumption will definitely increase
- Wages will go up as well: $zF_N(K, N) = w$
- What happens to labor supply?

Comparative statics: increase in TFP



Key questions

- What is the economic intuition behind the optimality condition $\text{MRS}_{l,C} = w$ for individual consumers? Explain why an individual maximizing welfare can do better when $\text{MRS}_{l,C} \neq w$
- How is the representative consumer's behavior affected by an increase in dividends income?
- What is the economic intuition behind the optimality condition $F_N(K, N) = w$ for the representative firm? Explain why a the firm could increase its profits if it chooses employment such that $F_N(K, N) \neq w$
- What does it mean to have convex preferences and technology?
- Define competitive equilibrium and Pareto optimality
- Discuss what would happen to consumption, leisure and total output when there is an exogenous increase in government expenditure in our 1-period model of the economy

References

- *Macroeconomics* (5th Edition) Stephen D. Williamson. Chapters 4 and 5.
- Kocherlakota, N. (2010): "Modern Macroeconomic Models as Tools for Economic Policy," The Region, Federal Reserve Bank of Minneapolis.